Math 300: Homework Set #5.

1. How many different ways are there to choose a baker’s dozen (13) donuts from 21 varieties at a donut shop?

2. A croissant shop has plain, cherry, chocolate, almond, apple, and broccoli croissants. How many ways are there to choose

(a) three dozen croissants?
(b) two dozen croissants with at least five chocolate croissants and at least three almond croissants?
(c) two dozen croissants with no more than two broccoli croissants?
(d) two dozen croissants with at least one plain croissant, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissants?

3. How many non-negative integer solutions are there to the equation

\[ x_1 + x_2 + x_3 + x_4 + x_5 = 21, \]

such that

(a) there are no further restrictions of \( x_1, \ldots, x_5 \)?
(b) \( x_i \geq 2 \) for each \( i = 1, 2, 3, 4, 5 \)?
(c) \( 0 \leq x_1 \leq 10 \)?

4. Find the number of non-negative integer solutions to \( x_1 + x_2 + x_3 \leq 11 \).

**Hint:** Why is it equivalent to find the number of non-negative integer solutions to \( x_1 + x_2 + x_3 + x_4 = 11 \)?

5. How many strings of 20-decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s, and three 9s?

6. Find the coefficient of \( x^3y^2z^5 \) in \((2x - 3y + 5z)^{10}\).

7. How many terms are there in the expansion of \((x + y + z)^{100}\)?