Math 351: The Third Homework Set.

Please answer at least 4 of the 5 questions below.

1. Find the supremum and infimum of \( \{x \in [0, 1] : x \notin \mathbb{Q} \} \). Prove why your assertions are correct.

2. Prove that \( \left\{ \frac{a}{2^b} : a \in \mathbb{Z}, b \in \mathbb{Z}_{\geq 0} \right\} \) is dense in \( \mathbb{R} \). (That is, show that we can find such a fraction between any two given real numbers.)

3. If \( A, B, C, D \) are sets such that \( A \sim B \) and \( C \sim D \), then there exist bijections \( f : A \to B \) and \( g : C \to D \). Define \( h : A \times C \to B \times D \) by \( h(a, c) = (f(a), g(c)) \). Show that \( h \) is a bijection (and thus \( A \times C \sim B \times D \)).

4. Use the \( \epsilon \)-definition of convergence to show that \( \lim_{n \to \infty} \frac{3n + 1}{2n + 5} = \frac{3}{2} \).

5. Argue that the sequence \( \{1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, \ldots\} \) does not converge to 0. (Hint: Can you find a value of \( \epsilon > 0 \) that there is no \( N \in \mathbb{N} \) so that the inequality in the definition of convergence is valid?)