1. Let $a_n \geq 0$ for all $n \in \mathbb{N}$.
   (a) If $\{a_n\}$ converges to 0, then $\{\sqrt{a_n}\}$ converges to 0.
   (b) If $\{a_n\}$ converges to some $x > 0$, then $\{\sqrt{a_n}\}$ converges to $\sqrt{x}$.

2. (a) Prove that $|a - b| \geq ||a| - |b||$ for any $a, b \in \mathbb{R}$.
   (b) If $\{b_n\}$ converges to $b$, then prove that $\{|b_n|\}$ converges to $|b|$.
   (c) If $\{|b_n|\}$ converges, then does $\{b_n\}$ converge? Explain.

3. Suppose that $\{a_n\}$ converges to 0, and $\{b_n\}$ is bounded (which means that there exists $M > 0$ such that $|b_n| < M$ for all $n \in \mathbb{N}$). Prove that $\{a_nb_n\}$ converges to 0.

4. Prove at least one of the following with the Squeeze Theorem.
   (a) $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{5^2}, \frac{1}{2^5}, \frac{1}{7^2}, \frac{1}{2^7}, \ldots\}$ converges to 0.
   (b) Prove that if $a, k > 0$, then $\lim_{n \to \infty} \frac{k}{n^k} \cdot \frac{n}{a} = \frac{1}{a^k}$.

5. Prove the Monotone Convergence Theorem in the case that the sequence $\{a_n\}$ is decreasing.

6. Let $r$ be a rational number greater than 1. Define the sequence $\{x_n\}$ by $x_1 = 1$ and
   $x_{n+1} = \frac{r(x_n + 1)}{x_n + r}$ if $n > 1$.
   (a) Show that $1 < x_n^2 < r$ for all $n > 1$.
   (b) $x_n < x_{n+1}$ for all $n \in \mathbb{N}$.
   (c) Conclude that this sequence has a limit, and show that it converges to $\sqrt{r}$.

7. By using the Monotone Convergence Theorem, show that $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.