Math 351, The Sixth Homework Set.

1. Let \( E = \{ r \in \mathbb{Q}_{\geq 0} \mid r^2 < 7 \} \).
   (a) Determine whether \( E \) is an open set as well as a closed set. Explain.
   (b) Find the limit points of \( E \). Justify your answer.

2. The following sets are not compact. For each set, provide a sequence (without proof) contained in the given set that does not possess a subsequence converging to a limit in the set.
   (a) \( \mathbb{Q} \cap [0, 1] \)
   (b) \( \mathbb{R} \)
   (c) \( \{ \frac{1}{n} \mid n \in \mathbb{N} \} \)

3. Find an open cover \( \{ \frac{1}{n} \mid n \in \mathbb{N} \} \) which does not have a finite subcover.

4. Prove that if \( K \) is compact, then sup \( K \) and inf \( K \) both exist and are elements of \( K \).

5. Without using the Heine-Borel Theorem, show that a closed subset \( A \) of a compact set \( K \) is compact.

6. Use the \( \delta - \epsilon \) definition of a limit to establish each of the following.
   (a) \( \lim_{x \to 2} (3x^2 - 5x + 1) = 3 \)
   (b) \( \lim_{x \to 9} \sqrt{x} = 3 \)

7. Give \( \delta - \epsilon \) definitions for the one-sided limits \( \lim_{x \to c^-} f(x) = L \) and \( \lim_{x \to c^+} f(x) = M \).