Math 352, Review for the First Exam.

Here are a few other exercises (in addition to the homework problems).

1. If 3 balls are randomly drawn without replacement from a bowl containing 6 white and 5 black balls, what is the probability that
   (a) one of the drawn balls is white and the other two are black?
   (b) at least one ball of each color is drawn?

2. In a certain community, 36% of the families own a dog, and 22% of the families that own a dog also own a cat. In addition, 30% of the families own a cat.
   (a) What is the probability that a randomly selected family owns a dog or a cat?
   (b) What is the conditional probability that a randomly selected family owns a dog given that it owns a cat?

3. Suppose that a random variable $Y$ is equal to the number of hits obtained by a certain baseball player in his next 3 at bats. If $P(Y = 1) = 0.3$, $P(Y = 2) = 0.2$, and $P(Y = 0) = 3P(Y = 3)$, find $E[Y]$ and $\text{Var}(Y)$.

4. You ask your neighbour to water a sickly plant while you are on vacation. Without water, it will die with probability 0.8; with water it will die with probability 0.15. You are 90% certain that your neighbour will remember to water the plant.
   (a) What is the probability that the plant will be alive when you return?
   (b) If it is dead, what is the probability that your neighbour forgot to water it?

5. Suppose that $A, B$ are disjoint, and $B, C$ are disjoint. If $P(A) = 0.3$, $P(B) = 0.2$, $P(C) = 0.4$, and $P(A \cup C) = 0.5$, find $P(A | B \cup C)$.

6. Suppose that $P(X = 0) = 1 - P(X = 1)$, and $E[X] = 3 \text{Var}(X)$. Find $P(X = 0)$.

7. Show that $P(A \cup B | C) = P(A | C) + P(B | C) - P(A \cap B | C)$.

8. Suppose that M&M’s have colours which satisfy a multinomial distribution: 30% Brown, 20% Yellow, 20% Red, 10% Orange, 10% Green, and 10% Blue.
   (a) What is the probability that if 10 M&M’s are randomly chosen, then exactly 6 of them are green, 2 are red, 1 is blue, and 1 is yellow?
   (b) What is the probability that if 10 M&M’s are randomly chosen, then at least 8 of them are brown? (Can you think of this as an equivalent binomial distribution?)

9. Suppose that a continuous random variable has a distribution whose pdf is $p(x) = cx^2e^{-7x^3}$ for $x \geq 0$, and 0 otherwise. What is the value of the constant $c$?

10. Be able to distinguish the various discrete distributions as discussed in class.
Solutions:

1. (a) \( \binom{6}{1} \binom{5}{2} \binom{1}{1} \binom{11}{3} \), (b) \( \binom{6}{1} \binom{5}{2} + \binom{6}{2} \binom{5}{1} \binom{11}{3} \)

2. (a) \( 0.36 + 0.30 - 0.22 = 0.44 \), (b) \( P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{0.22}{0.30} \)

3. First of all, find that \( P(0) = 0.375 \) and \( P(1) = 0.125 \). Then, \( E[Y] = 1.075 \) and \( E[Y^2] = 2.225 \). So \( \text{Var}(Y) = 1.069375 \).

4. (a) \( P(A) = P(A|W)P(W) + P(A|\overline{W})P(\overline{W}) = (1 - .15)(.9) + (1 - .8)(.1) = .785 \).
   (b) \( P(W|\overline{A}) = \frac{P(\overline{W} \cap \overline{A})}{P(\overline{A})} = \frac{(0.1)(0.8)}{1 - .785} = \frac{16}{43} \).

5. By Inclusion-Exclusion, \( P(A \cup C) = P(A) + P(C) - P(A \cap C) \) implies that \( P(A \cap C) = 0.2 \), and \( P(B \cup C) = P(B) + P(C) - 0 = 0.6 \).
   So, \( P(A|B \cup C) = \frac{P(A \cap (B \cup C))}{P(B \cup C)} = \frac{P(A \cap C)}{P(B) + P(C)} = \frac{1}{3} \).

6. Let \( n = P(X = 0) \) so that \( P(X = 1) = 1 - n \). So, \( E[X] = 0P(0) + 1P(1) = 1 - n \) and \( E[X^2] = 0^2P(0) + 1^2P(1) = 1 - n \). Substituting this into \( E[X] = 3 \text{ Var}(X) \) yields \( n = \frac{1}{3} \) or 1.

7. Use the definition of conditional probability along with the Inclusion-Exclusion principle to prove this.

8. (a) Use a multinomial distribution: \( \frac{10!}{6!2!1!1!} \cdot (0.1)^6(0.2)^2(0.1)^1(0.2)^1 \).
   (b) Think of a binomial distribution with \( p = 0.3 \) (brown probability) and so \( q = 0.7 \) (other colours). So, the desired probability equals
   \( P(8) + P(9) + P(10) = \binom{10}{8} (0.3)^8(0.7)^2 + \binom{10}{9} (0.3)^9(0.7) + \binom{10}{10} (0.3)^{10} \).

9. Solve \( \int_0^\infty cx^2e^{-7x^3} \, dx = 1 \) (with the substitution \( w = -7x^3 \)) to find that \( c = 21 \).