1. Know the definitions of a group (what are the four axioms?), and how to test for a subgroup (two axioms to check). What is the order of an element/subgroup?

2. Examples of groups:
   
   (a) Cyclic groups (finite: $\mathbb{Z}_n$ and $U_n$, and infinite: $\mathbb{Z}$) and their basic properties (structure of subgroups, locating generators, ...)
   
   (b) Nonabelian groups (especially $D_n$ and $S_n$) and how to work with them.
   
   (c) Miscellaneous groups ($K_4$, $Q_8$, matrix groups, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, etc.)

3. Know these proofs:
   
   - Cancellation Laws.
   - Uniqueness of identity and inverses.
   - A subgroup of a cyclic group is cyclic.
   - Cyclic groups are abelian.
   - $(ab)^{-1} = b^{-1}a^{-1}$ for any $a$ and $b$ from a group.
   - Groups of prime order are cyclic.

4. Sample questions:
   
   (a) What is the identity element for the binary operation $*$ on $\mathbb{R}$ defined by $a * b = 2ab + a + b$? Do inverses exist?
   
   (b) Let $H$ and $K$ be groups. Show that $H \times K = \{(a,b) : a \in H \text{ and } b \in K\}$, equipped with componentwise multiplication $(a,b) \cdot (c,d) = (ac, bd)$, is also a group (this is called the “direct product” of $H$ and $K$).
   
   (c) Compute the orders for the following elements in their respective groups.
      
      - (a) $-i \in \mathbb{C}$, (b) $(123)(45) \in S_6$, (c) $8 \in \mathbb{Z}_{36}$, (d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \in GL_2(\mathbb{R})$
   
   (d) Show that $\{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is a subgroup of $\mathbb{R}$.
   
   (e) What is the smallest noncyclic abelian group? Smallest nonabelian group?
   
   (f) Is the union of two groups a group? Prove it or give a counterexample.
   
   (g) Let $G$ be the group with composition as the binary operation generated by the two elements $\frac{1}{2}$ and $\frac{\pi-1}{2}$. Create the multiplication table for this group (you should find $|G| = 6$). To which group is this isomorphic? Explain.