1. Find all solutions to $f(x) + 2f\left(\frac{1}{x}\right) = 3x^2$.

2. Suppose that $\{u_n\}, \{v_n\}, \text{ and } \{w_n\}$ are real-valued sequences which satisfy the two relations $u_n + v_n + w_n = 3$ and $u_n^2 + v_n^2 + w_n^2 = 3$ for all $n$.
   If the limits exist, find $\lim_{n \to \infty} u_n$, $\lim_{n \to \infty} v_n$, and $\lim_{n \to \infty} w_n$.

3. Recall the Fibonacci Sequence $\{F_n\}$ is defined by $F_1 = F_2 = 1$ and $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 0$. (We will also find it useful to define $F_0 = 0$.)
   Show that if $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^n = \begin{bmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{bmatrix}$ for all $n \in \mathbb{N}$.
   \textbf{Note}: This relation can often be used to prove Fibonacci identities.

4. Compute $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n + 1)}{n^2 + n}$.

5. Consider $y = f(x)$, where $f(0) = f\left(\frac{1}{n}\right) = 0$ for all $n \in \mathbb{N}$. If the graph of $f(x)$ on $[0,1]$ consists of the congruent sides of an isosceles triangle of height 1 for each $n \in \mathbb{N}$, compute $\int_0^1 f(x) \, dx$.

6. Compute $\lim_{x \to 1} \frac{\sin(\sqrt{x+3} - 2)}{\tan(\sqrt{x^2 + 5x + 3} - 3)}$.

7. Let $P(x)$ be a polynomial of degree 2012 such that $P(k) = \frac{1}{k}$ for all $k = 1, 2, ..., 2013$.
   What is $P(0)$?
   \textbf{Hint}: First, consider $Q(x) = xP(x) - 1$.

8. For any $n > 0$, evaluate $\int_0^{\infty} [x]e^{-nx} \, dx$.

9. Show that $14x^2 + 15y^2 = 7^{2012}$ has no integer solutions.
   \textbf{Hint}: Show that if such a solution exists, then $y$ is divisible by 7. Then, show that $x$ is also divisible by 7. What happens to the equation after this?