Math 300: Review for the Final Exam.

For additional practice, please look at previous exams and homework.

1. How many ways can you
   
   (a) form 5-digit octal (base 8) strings ending in '0'?
   (b) arrange the five (distinct) regular polyhedra dice?
   (c) park 10 cars in a lot with 18 spaces?
   (d) pick a committee of 12 people from 100 senators?
   (e) arrange 30 M&M's if 9 are brown, 6 are yellow, 3 are red, 3 are orange, 7 are green, and 2 are blue?
   (f) distribute two dozen mushrooms among Frodo, Sam, Merry, and Pippin?

2. A donut store which specialises in making buttermilk donuts sells buttermilk donut bars, pumpkin buttermilk donuts, and strawberry buttermilk donuts. How many ways can you purchase one dozen buttermilk donuts if:
   
   (a) there are no restrictions on any particular type of donut?
   (b) at least 5 bars?
   (c) at least 5 bars, between 1 and 4 (inclusive) strawberry?

3. Let A and B be sets with m and n elements, respectively.

   (a) How many functions \( f : A \to B \) are there?
   (b) If \( m \leq n \), then how many functions \( f : A \to B \) are 1-1?
   (c) If \( m \geq n \), then how many functions \( f : A \to B \) are onto?

4. Use a generating function to solve the following problem:

   A bakery specialises in making the following types of delicious scones: apple, orange, cinnamon, cherry, blueberry, and honey. How many ways can I purchase 20 scones if I buy an even number of apple scones, a multiple of 3's worth of cinnamon scones, at most 1 orange scone, at most 2 cherry scones, and at least 4 honey scones?

5. How many positive integers less than or equal to 1000 are multiples of 8, 12, or 15?

6. Prove by Mathematical Induction:

   (a) \( \sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1} \).
   (b) \( \sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3} \).
(c) A chocolate bar consists of \( n \) squares arranged in a rectangular pattern. If you split the bar into small squares by breaking along the lines between the squares, what is the minimum number of breaks?

(d) Let the “Tribonacci sequence” be defined by \( T_1 = T_2 = T_3 = 1 \) and 
\[ T_n = T_{n-1} + T_{n-2} + T_{n-3} \] 
for all \( n \geq 4 \). Prove that \( T_n < 2^n \) for all \( n \in \mathbb{N} \).

7. Solve the recurrences:

(a) \( a_n = 2a_{n/2} + 7 \), with \( a_1 = 0 \) (Let \( n = 2^k \) and use back substitution.)
(b) \( a_{n+2} - 4a_{n+1} + 4a_n = 8 \) with \( a_0 = 3 \) and \( a_1 = 5 \).
(c) \( 2a_{n+4} + a_{n+3} - 15a_{n+2} - 23a_{n+1} + 15a_n = 0 \).

8. Create a recurrence for the following problem (and solve it): How many words of length \( n \) can be formed with the condition that the letter \( Z \) never appears after an \( A \)? (Partial answer: \( a_n = 25a_{n-1} + 25^{n-1} \).

9. Given the following two graphs:

(a) Calculate the degree of each vertex.
(b) Verify the Handshake Theorem.
(c) Does it contain an Euler cycle or path?

10. Determine the chromatic number of \( W_n \) for \( n \geq 3 \). (There are two cases: \( n \) is even or odd...