Radio Labeling Grid Graphs

CSU CI Mathematics REU

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What is Radio Labeling?

The distance between two vertices u and v, $d(u,v)$, is the length of the shortest path between u and v.

The diameter of a graph G, diam(G), is the maximum distance in a graph, taken over all pairs of vertices.

A radio labeling is a function c that assigns positive integer values (labels) to vertices so as to satisfy the *radio condition*

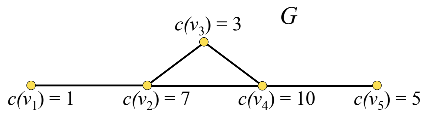
$$d(u,v) + |c(u) - c(v)| \geq \text{diam}(G) + 1.$$

The span of a radio labeling c, *span*(c), is the maximum integer assigned by c to a vertex in G.

The radio number of G, $rn(G)$, is the minimum achievable span.

Ex:

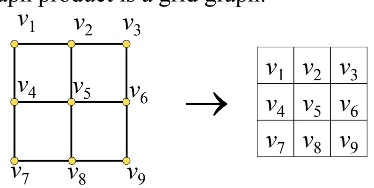
$d(v_2,v_5) = 2$
diam$(G) = d(v_1,v_5) = 3$
span$(c) = 10$



Grid Graphs

When we "multiply" two path graphs (take the Cartesian product), the graph product is a grid graph.

Ex: $P_3 \square P_3$



Vertices are now represented by boxes; two boxes represent adjacent vertices if they share an edge.

Strategy: Upper Bound for $rn(P_n \square P_n)$

1. Specify the order in which to label vertices. (We do this in stages, as indicated in the center diagrams.)
2. Give the vertices the minimum label values required so as to satisfy the radio condition.
3. Calculate the span of this labeling.

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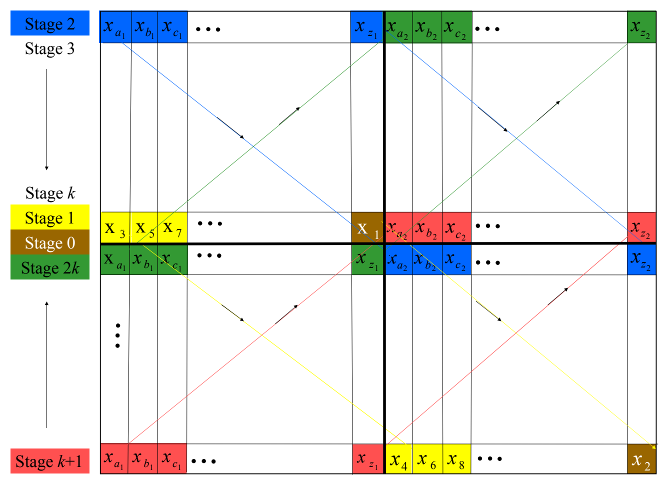
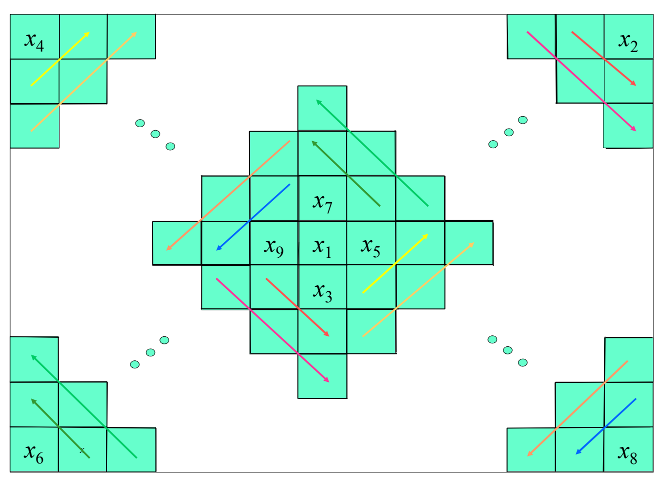
Our Two Labeling Algorithms

Evens



Odds



Analyze distances between consecutively-labeled vertices and increment label values to satisfy the radio condition.

Strategy: Lower Bound for $rn(P_n \square P_n)$

Let c be any radio labeling of $P_n \square P_n$.

1. Develop an equation relating span(c) to the sum of distances between consecutively-labeled vertices.
2. Minimize the span by maximizing the sum of distances.

$$span(c) \geq (n^2-1)[\text{diam}(G)+1]+1-\sum_{i=1}^{n^2-1} d(x_i,x_{i+1})$$

$$\sum_{i=1}^{n^2-1} d(x_i,x_{i+1}) = d(x_1,x_2)+\cdots+d(x_{n^2-1},x_{n^2})$$

$$\text{So } \sum_{i=1}^{n^2-1} d(x_i,x_{i+1}) = |\sigma(1)-\sigma(2)|+|\tau(1)-\tau(2)|+$$
$$+\ |\sigma(2)-\sigma(3)|+|\tau(2)-\tau(3)|+\cdots$$
$$+|\sigma(n-1)-\sigma(n)|+|\tau(n-1)-\tau(n)|$$

Odds

$$rn(P_n \square P_n) \geq ((2k+1)^2-1)[\text{diam}(G)+1]+1-\sum_{i=1}^{(2k+1)^2-1} d(x_i,x_{i+1})$$

$$4(2k+1)\Big[\sum_{i=k+2}^{2k+1} i - \sum_{i=1}^{k} i\Big]$$

Evens

$$rn(P_n \square P_n) \geq ((2k)^2-1)[\text{diam}(G)+1]+1-\sum_{i=1}^{n^2-1} d(x_i,x_{i+1})$$

$$(4)2k\Big[\sum_{i=k+2}^{2k} i - \sum_{i=1}^{k-1} i\Big]+\big[(k+1)(8k-2)-k(8k-2)\big]$$

Our Results

Upper and Lower Bounds:

$$n \text{ even}: \quad n^3-n^2-2n+4 \leq rn(P_n \square P_n) \leq n^3-n^2+1$$

$$n \text{ odd}: \quad n^3-n^2-n+2 \leq rn(P_n \square P_n) \leq n^3-n^2-\frac{(n-1)}{2}+1$$

References

[1] G. Chartrand, D. Erwin, and P. Zhang Radio Labelings of Graphs, Bulletin of the ICA, **33,** (2001),77-85.

[2] D. Liu, and X. Zhu, Multilevel distance labelings for paths and cycles, SIAM J. Discrete Math. **19** (2005), No. 3, 610-621.

Research Mentors

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