An operator is a linear function defined on a vector space. If the vector space has a norm and it is complete with respect to that norm, we say that the space is a Banach space. Modern Operator Theory deals (mostly) with properties of operators on Banach spaces (mostly infinite dimensional) and trying to somehow “decide” if an operator has or does not have certain properties. One important property is that of being bounded.

An operator $T$ on a Banach space $B$ (with norm $\| \cdot \|$) is bounded if there exists a constant $M$ such that $\|Tx\| \leq M\|x\|$ for every $x \in B$. Boundedness is important for several reasons, perhaps the most important being that an operator is bounded if and only if it is continuous.

Another type of property studied in Operator Theory is a dynamical property: an operator $T$ on a Banach space $B$ is hypercyclic if there exists a vector $x \in B$ such that the orbit

$$\text{orb}(T,x) := \{ x, Tx, T^2x, T^3x, \ldots \}$$

is dense in $B$. Here $T^n$ denotes the composition of $T$ with itself $n$ times; that is,

$$T^n := T \circ T \circ \ldots \circ T \quad \text{n times}$$

It is perhaps surprising that such operators exist (they do, but only in spaces of infinite dimensions). Their existence is related to several problems, including that of deciding if an operator is chaotic.

On the other hand, given a discrete object, for example, an infinite tree, one can consider the space of functions whose domain is the object. The set of such functions is naturally a vector space and one can give this space a norm (in fact, several different norms) which makes the vector space a Banach space.

In this research group, we will study properties of different operators on Banach spaces on trees and attempt to relate the properties of the operator with the structure of the tree. Some of the properties of operators that we will study are boundedness, compactness, finite-rank operators, isometries, eigenvalues and spectrum and dynamical properties such as cyclicity and hypercyclicity. To give a taste of the type of results we hope to obtain, in [5], it was shown that a “backward shift” operator is hypercyclic if and only if the tree has no “free ends”. Many other such results appear in the references.

A student interested in this project should have necessarily completed a Linear Algebra course and, preferably, a Real Analysis course. Beyond that, I will assign some basic bibliography for the students to read before arriving to CSUCI.

References


