Fractal signature and lacunarity in the measurement of the texture of trabecular bone in clinical CT images

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Abstract

Fractal analysis is a method of characterizing complex shapes such as the trabecular structure of bone. Numerous algorithms for estimating fractal dimension have been described, but the Fourier power spectrum method is particularly applicable to self-affine fractals, and facilitates corrections for the effects of noise and blurring in an image. We found that it provided accurate estimates of fractal dimension for synthesized fractal images. For natural texture images fractality is limited to a range of scales, and the fractal dimension as a function of spatial frequency presents as a fractal signature. We found that the fractal signature was more successful at discriminating between these textures than either the global fractal dimension or other metrics such as the mean width and root-mean-square width of the spectral density plots. Different natural textures were also readily distinguishable using lacunarity plots, which explicitly characterize the average size and spatial organization of structural sub-units within an image. The fractal signatures of small regions of interest (32 × 32 pixels), computed in the frequency domain after corrections for imaging system noise and MTF, were able to characterize the texture of vertebral trabecular bone in CT images. Even small differences in texture due to acquisition slice thickness resulted in measurably different fractal signatures. These differences were also readily apparent in lacunarity plots, which indicated that a slice thickness of 1 mm or less is necessary if essential architectural information is not to be lost. Since lacunarity measures gap size and is not predicated on fractality, it may be particularly useful for characterizing the texture of trabecular bone.

Keywords: Texture; Fractal dimension; Fractal signature; Lacunarity; Bone architecture

1. Introduction

Fractal models have long been considered appropriate for modelling the texture in medical images, with fractal dimension commonly used as a compact descriptor. The fractal dimension describes how an object occupies space and is related to the complexity of its structure: it gives a numerical measure of the degree of boundary irregularity or surface roughness. Exact fractals have attractive properties, such as invariance to scale and projection: but for real structures, fractality is present only in a statistical sense and only over a limited range of scales. The estimation of fractal dimension is a notoriously difficult procedure, complicated by the fact that the values (both elevation and position) for real data are digitized and are often sparse and cover only a relatively short range of dimensions.

Numerous algorithms for estimating fractal dimension have been described [1–7]. They are all based on measuring an image characteristic, chosen heuristically, as a function of a scale parameter. Generally these two quantities are linearly regressed on a log–log scale, and the fractal dimension obtained from the resulting slope, although nonparametric estimation techniques have also been used [8]. However, the image characteristic of interest must be chosen with care if the resulting estimate is to be a valid, reliable and accurate indicator of fractal dimension [9,10]. Certain characteristics can be less robust when applied to digitized data, especially when these are sparse. Algorithms that implicitly assume an exactly self-similar fractal model are inappropriate for medical images, since in particular pixel intensity and position are different physical properties and cannot be expected to scale with the same ratio. Thus, methods that...
do not meet the intensity scale independency requirement [11], such as the blanket [2], box-counting [6] and area [7] algorithms, were not considered. In contrast, the Fourier power spectrum method conveniently represents the statistical nature of real images by describing them in terms of a fractional Brownian motion model [12]. It has been shown to estimate the fractal dimension of self-affine fractals reliably and accurately [13,14]. The variation of fractal dimension with scale can be considered as the fractal signature. The concept of a fractal signature has previously been used in the spatial domain to distinguish Brodatz textures [2] and textures in conventional radiographs of osteoarthritic knees [15] and lumbar vertebrae [16].

Lacunarity is a multi-scale measure of texture describing the complex intermingling of the shape and distribution of gaps within an image; specifically, it quantifies the deviation of a geometric shape from translational invariance. It is not predicated on self-similarity (i.e., fractality), and has been used most successfully with binarized images [17,18]. A plot of a lacunarity against window size contains significant information about the spatial structure of an image at different scales. In particular, it can distinguish varying degrees of heterogeneity within an image, and in the case of a homogeneous image it can identify the size of a characteristic substructure. Lacunarity has been used previously to characterize landscape texture in binarized optical [17] and SAR (synthetic aperture radar) [18] images.

Osteoporosis is a prevalent bone disease characterized by a debilitating loss of bone strength and, consequently, fracture risk and the spine is a useful site for predicting osteoporotic failures. Although the relative contributions of trabecular and cortical bone to overall bone strength are unclear [19], most studies have concentrated on trabecular bone since it is the metabolically more active as evidenced by remodelling data [20]. It has been increasingly recognized that quantifying the structural quality of vertebral trabecular bone (assessed in terms of the integrity of its internal architecture) may assist in an earlier and more accurate diagnosis of osteoporosis than assessing bone quantity [in terms of bone mineral density (BMD)] alone [21,22]. Indeed, fractal analysis of high-resolution images has been used to characterize bone microarchitecture [23]: and the fractal dimension of cubes of vertebral trabecular bone has been shown to be highly correlated with their elastic modulus [24]. A major advantage of computed tomography (CT) imaging over other modalities such as dual energy X-ray absorption (DEXA) is its ability to isolate and measure trabecular bone separately from cortical bone. Since this is a prerequisite for texture studies CT imaging would be required for patient studies, even though the patient dose is higher than for a DEXA examination [25]. Although the limited resolution of commercial CT scanners precludes proper resolution of the trabecular structure 2-D axial images of thin vertebral slices contain some of this architectural information, albeit degraded by the inadequate modulation transfer function (MTF) of the imaging system, which we will refer to as texture.

The purpose of this study was to investigate the potential usefulness of fractal signature and lacunarity in quantifying the texture of trabecular bone in clinical CT images. Because of the coarse raster of clinical CT scanners the trabecular bone images used were small, typically 32×32 pixels. We used synthetic fractal images to assess the accuracy of our algorithm for estimating the fractal dimension of data sets of limited resolution. Natural texture images were used to determine the extent of fractality in real images, and to assess the efficacy of a fractal signature in discriminating between different textures. These images were also used to validate the lacunarity methodology and subsequent interpretation of lacunarity plots. The two methods, fractal signature and lacunarity, were then applied to small trabecular bone CT images to assess their utility and sensitivity in quantifying texture therein. The significance of correcting for the limited resolution and noise of the imaging system was also addressed.

2. Materials and methods

2.1. Test images

Two sets of test images were used to test the accuracy of our protocols and their subsequent interpretation.

1. Synthetic fractal images (64×64 pixels) were generated with fractal dimensions, D, of 2.25 to 4.0 in steps of 0.25, by filtering a white noise image in a method similar to that described by Saupe [4].

2. Photographic images of texture from an album by Brodatz [26] have become a de facto standard for testing texture algorithms. We used a subset of the textures used in earlier studies [2,5]: pressed cork (D04), grass lawn (D09), beach sand (D28) and pigskin (D92) (Fig. 1). The textures were digitized to produce images of 512×512 pixels with 8-bit depth.

2.2. CT bone images and scanner MTF

The lumbar vertebrae (L4) from five male human donors (mean age 43±3 yr) were collected at autopsy.

Fig. 1. Digitized texture images: (from left to right) pressed cork, grass lawn, beach sand, pigskin. (After Brodatz [26]).
All had died suddenly from accidents or from acute diseases: none had a history of metabolic bone disease or bone fractures. The vertebrae were soaked in a dilute solution of formalin for one week to rehydrate the bone and eliminate trapped air from the trabeculae. Axial images of the vertebrae were obtained using a GE Sytec 3000 CT scanner (General Electric, Milwaukee, WI, USA) at 100 kVp and 100–130 mA. Scans were taken through the middle of each vertebra, parallel to the vertebral endplates, with slice thicknesses of 1, 3, 5 and 10 mm. The scan time was 3 s per view, the scan field of view (FOV) was 25 cm, and the supplied bone kernel was used in the reconstruction to duplicate clinical scanning conditions. A standard QCT phantom (Image Analysis Inc., Irvine, CA, USA), comprising water and two different densities of mineral-equivalent calcium hydroxyapatite (75 and 150 mg cm⁻³), was included in each view. The mean bone mineral density (BMD) of the vertebrae (230±20 mg cm⁻³) was within the normal clinical range.

The point spread function (PSF) of the CT scanner was measured by scanning thin (0.05 mm diameter) tungsten wires in a cylindrical container of water positioned perpendicular to the scan plane. Three wires were used in each scan, and the mean profile was averaged about its peak [27] to give a circularly symmetric PSF whose Fourier transform was the required MTF of the system. The full width at half maximum height (FWHM) of the PSF, which corresponds to the in-plane resolution of the images, was 1.11 mm under our scanning conditions.

2.3. Determination of power spectra and fractal signature

The relationship of the fractal dimension to the Fourier power spectrum, as a consequence of using fractional Brownian motion as a model for natural fractals, is addressed in Appendix A.

Each of the five vertebrae resulted in a series of four images acquired with differing slice thicknesses. For each image within a series, a square (32×32 pixels) region of interest (ROI) within the trabecular bone and close to the anterior surface of the vertebra was extracted (Fig. 2) using the pixel coordinates of the images to select identically registered ROIs. For each image another 32×32 pixel ROI (not shown), from the region corresponding to the water phantom, was also extracted to characterize the additive noise in the imaging system. A two-dimensional Fourier power spectrum of each (32×32) image was obtained, from which a radial power spectrum was generated by averaging values over increasingly larger annuli for each of the radial increments.

The images of the bone are degraded by noise and blurring within the CT scanner, which affect the power spectrum and hence the estimate of fractal dimension. The effect of noise is to add additional roughness to an image resulting in an overestimate of fractal dimension: when attempting to measure the fractal dimension, D, of smooth surfaces (D=2) the presence of even small amounts of noise can be confounding [3]. The noise power depends on exposure conditions, and varies between different scanners. It will have a proportionately larger effect at higher spatial frequencies, leading to a flattening effect on the image power spectrum. Consequently, any attempt to correct image spectra by subtracting a constant ‘white noise floor’ [28] is inappropriate. Instead, a correction for additive system noise for particular scan conditions was attempted using the water sample included in the QCT phantom. The Fourier transform of the water image gives the power spectrum of the additive noise, which is also present in the bone image. Since both the water and the bone image were collected under the same exposure conditions (except for beam-hardening effects), subtraction of the water (i.e. noise) spectrum from the bone spectrum may reduce the effect of system noise. Spectral subtraction has been used to reduce noise in speech processing [29] and in scanning tunnelling microscopy images [30]. The power spectrum method for estimating fractal dimension is the only algorithm that allows such subtraction to be implemented easily.

Image blurring within an imaging system can be described by the system MTF. For CT this includes the effects of quantization and possible aliasing, and the algorithm used in the image reconstruction process. The consequences of these effects on the image have been modelled [27]. The overall result is to progressively filter out the higher frequencies of the object, resulting in a steeper power spectrum and a consequent underestimate for the fractal dimension. The measured power spectrum of the image, S′(ω), is given by

\[ S'_\omega(\omega) = S_\omega(\omega) \times (MTF)^2 \]  

(1)

where S(ω) is the actual power spectrum of the object. The effect of system blurring can therefore be eliminated by dividing the measured power spectrum by the square of the MTF. Only the power spectrum method is amenable to such a straightforward correction for MTF blurring. By implementing corrections for the effects of additive system noise and the modulation transfer func-
tion (MTF) we were able to obtain fractal dimension estimates that were independent of the CT scanner used and its settings.

For each image, the radial power spectrum was plotted on a log–log scale after correction for both additive noise and MTF blurring, and the local fractal dimension was obtained from the local slope, $\beta$ (within overlapping windows of spatial frequency), using

$$D = 4 - \beta/2$$  \hspace{1cm} (2)

We will refer to the variation of local fractal dimension with spatial frequency as the fractal signature of the image. Some studies [31,32] exclude the first few terms in fitting a line to the power spectrum, whilst others [33] suppressed their effect by filtering the spectrum by the visual system response of a human observer. We find this latter scheme arbitrary, in view of the variable parameters that were used to shift the peak of this (band-pass) filter, and followed the former scheme.

The mean width of the radial power spectrum (viz., the average spatial frequency of the image) has been used as a texture measure in the detection of interstitial lung disease on chest radiographs [33] and the analysis of trabecular texture in conventional spine radiographs [34]. We considered both the mean width and the root-mean-square (RMS) width of the power spectrum to determine whether either was a useful texture measure.

### 2.4. Lacunarity analysis

Lacunarity refers to the distribution of gap sizes in data: the greater the range in gap size distribution, the more lacunar the data [35]. An efficient algorithm for lacunarity estimation analyzes deviations from translational invariance of an image’s brightness distribution using gliding-box sampling [36]. The gliding-box exhaustively samples an image using overlapping square windows of length $r$. Lacunarity can be defined in terms of the local first and second moments, measured for each neighbourhood size, about every pixel in the image [18], i.e.

$$L(r) = 1 + \{\text{var}(r)/\text{mean}^2(r)\}$$  \hspace{1cm} (3)

where mean $(r)$ and var $(r)$ are the mean and variance of the pixel values, respectively, for a neighbourhood size $r$. The lacunarity was measured from the binarized ROIs as is common practice [37].

Lacunarity is sensitive to both the image density and its spatial configuration [18]. Sparse distributions have higher lacunarities than dense distributions given the same sampling window size and higher lacunarity values indicate more clumping, for any given image density and sampling window size. The lacunarity index as a function of sampling window is generally presented as a log–log plot, which illustrates the scale dependency of spatial nonstationarity in the image. The use of a square sam-

pling window constrains the manifestation of spatial heterogeneity to translational invariance. Higher lacunarity values represent a wider range of sizes of structures within an image. The decay pattern of the lacunarity plot contains significant information about the spatial structure of the image [17,18]. A spatially random image decays rapidly to the minimum value. An image with self-similarity across a range of scales exhibits a linear decay, the slope of which provides an estimate of its fractal dimension within that range. For an image comprising a distribution of structures of a similar size, the lacunarity decay is slow until the sampling window, $r$, exceeds the size of these structures and is rapid thereafter.

A normalized index, $NL(r)$, can be calculated by combining the lacunarity index, $L(r)$, with the ‘complementary’ lacunarity index, $cL(r)$, obtained by calculating the lacunarity of the complemented image, i.e.

$$NL(r) = 2 - (1/L(r) + 1/cL(r))$$  \hspace{1cm} (4)

Normalizing the lacunarity in this way ensures that its value decays from 1 to 0, so that the shape of the decay can be compared independent of image density.

### 2.5. Statistical analysis

Data were expressed as the mean±standard deviation. Spearman’s correlation coefficient was used to express the degree of association between two variables. The unpaired Student’s $t$-test was used to compare means: the level for significance was taken as $P=0.05$. Statview 4.02, running on a Power PC/Macintosh, was used for the statistical analysis.

### 2.6. ROI size and intra-specimen variability

Digital images, by their nature, are limited by spatial and intensity quantization. The former especially has been shown to affect the estimation of fractal dimension [10]. Fractal dimension is more accurately estimated, especially for larger values, when the spatial resolution is high. In practice, due to the limited resolution of clinical CT scanners and the need for image sizes corresponding to an integral power of two for the fast Fourier transform (FFT) algorithm, we were unable to extract regions of interest containing trabecular bone larger than 32×32 pixels. Whilst this ROI was always positioned close to the anterior surface of the vertebrae in our protocol, we tested intra-specimen variability by selecting adjacent ROIs up to 4 pixels apart and found that the resulting estimates of fractal signature and lacunarity were not significantly different.
Table 1
Parameters for the synthesized fractal images using Fourier power spectrum analysis

<table>
<thead>
<tr>
<th>Theoretical $D$</th>
<th>Measured $D$ (±0.02)</th>
<th>Mean width$^a$</th>
<th>rms width$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>3.99</td>
<td>20.9</td>
<td>22.2</td>
</tr>
<tr>
<td>3.75</td>
<td>3.74</td>
<td>18.8</td>
<td>20.5</td>
</tr>
<tr>
<td>3.5</td>
<td>3.49</td>
<td>15.7</td>
<td>18.2</td>
</tr>
<tr>
<td>3.25</td>
<td>3.24</td>
<td>11.6</td>
<td>14.8</td>
</tr>
<tr>
<td>3.0</td>
<td>2.99</td>
<td>8.1</td>
<td>11.3</td>
</tr>
<tr>
<td>2.75</td>
<td>2.74</td>
<td>3.9</td>
<td>6.7</td>
</tr>
<tr>
<td>2.5</td>
<td>2.48</td>
<td>2.8</td>
<td>4.6</td>
</tr>
<tr>
<td>2.25</td>
<td>2.22</td>
<td>1.9</td>
<td>2.9</td>
</tr>
</tbody>
</table>

$^a$ The mean width and rms widths are given in units of cycles/image. To convert to units of cycles/pixel, the values should be divided by 64.

3. Results

3.1. Test images

The radial power spectral densities for the synthesized fractal images are linear (not shown), with statistical imprecision due to the finite size of the data sets [38]. Linear regression was used to obtain the gradients, and hence the measured fractal dimensions of the images. The close agreement of the measured and theoretical values of the fractal dimension of these images (Table 1) confirms the accuracy of the estimation algorithm. Both estimates of the width of the power spectrum, the mean width and the RMS width, correlate strongly with the fractal dimension ($r=0.987$ and 0.993 for the mean and RMS width, respectively, with the theoretical fractal dimension).

The power spectral density plots of the Brodatz test images are shown in Fig. 3. The data at low frequencies is unreliable due to the limited extent of the images but, beyond a log (frequency) of about 1.5, the textures appear well-behaved, although they exhibit some multifractality. The local slope over a limited range of spatial frequencies defines a fractal dimension at that scale, and the local fractal dimension as a function of spatial frequency then becomes the fractal signature of the image. The fractal signatures of the Brodatz test images (Fig. 4), displayed as functions of spatial frequency, are more successful at discriminating between different textures than the spectral density plots.

Whilst an earlier study [2] also showed the limited fractality of some of the Brodatz images, more recent studies [5] have computed only a global fractal dimension for each image. Table 2 compares reported values and the estimates we obtained by averaging over the mid-range of spatial frequencies. The different methods produce different values for the estimated global fractal dimension. However, the actual values of fractal dimension are less important than the relative values amongst a group of images if only differences amongst the images need to be recognized. The two indicators of power spectrum width, the mean width and the RMS width (Table 2), correlated well with our estimates of the fractal dimension (Spearman correlation coefficients of 0.951 and 0.944, respectively).

The normalized lacunarity plots for the Brodatz test images are shown in Fig. 5, for sampling windows ($r$) up to 100 pixels. The four images are easily distinguished. The beach sand image is the most lacunar, corresponding to the greatest relative clustering of structures within the image. The characteristic size of these structures corresponds to the neighbourhood size, after which decay of the lacunarity becomes rapid. A plot of the deviation of the lacunarity plots from linearity (viz., a self-similar fractal) emphasizes subtle features that

![Fig. 3. Radial power spectral density plots for the Brodatz test images. [The first 256 terms of the discrete Fourier transform (i.e. up to the Nyquist frequency) were used].](image)

![Fig. 4. Fractal signatures of the Brodatz test images. Local fractal dimension was calculated using overlapping windows of spatial frequency (terms 1–31, 17–47, 33–63 etc.), and plotted against the mean of the spatial frequency window.](image)
Table 2
Parameters for the Brodatz test images. (The estimated global fractal dimension, $D'$, is calculated over the mid-range (terms 64–192) of spatial frequencies. $D_1$, $D_2$, $D_3$ are estimates reported using the cell counting algorithm [5], the Fourier power spectrum algorithm [1], and the blanket algorithm [2] respectively)

<table>
<thead>
<tr>
<th>Image</th>
<th>$D'$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>Mean width</th>
<th>rms width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grass lawn</td>
<td>2.71</td>
<td>2.59</td>
<td>2.49</td>
<td>2.65</td>
<td>90.8</td>
<td>110.1</td>
</tr>
<tr>
<td>Pressed cork</td>
<td>2.59</td>
<td>2.66</td>
<td>2.55</td>
<td>2.72</td>
<td>76.5</td>
<td>92.8</td>
</tr>
<tr>
<td>Pigskin</td>
<td>2.51</td>
<td>2.50</td>
<td>2.38</td>
<td>2.59</td>
<td>72.2</td>
<td>92.3</td>
</tr>
<tr>
<td>Beach sand</td>
<td>2.42</td>
<td>2.55</td>
<td>2.48</td>
<td>2.61</td>
<td>45.4</td>
<td>65.6</td>
</tr>
</tbody>
</table>

Fig. 5. Normalized lacunarity of the Brodatz test images, $NL(r)$, plotted against the natural logarithm of the neighbourhood size, $r$. Neighbourhood sizes from 1 to 100 pixels were used.

may not be conspicuous in the lacunarity plots themselves: and a maximum in the deviation plot will indicate the characteristic size of structures within the image. A linear plot of lacunarity deviation against sampling window (not shown) indicates that this is about 7 pixels, which accords well with the typical size of a single grain of sand in the image. The lacunarity plot of the pressed cork image indicates some clustering of structures with a characteristic size of about 3 pixels which is consistent with a visual assessment of the image. The lawn grass image is the most homogeneous, and comprises small thin (~2 pixels across) objects, randomly distributed across the image. As a result, its lacunarity is the lowest. The lacunarity plot of the pigskin image indicates that it consists of larger structures (~7 pixels wide) that are less clustered than the structures in the cork image (but more than those in the grass image). This is consistent with a qualitative visual assessment. Although the size of these structures is almost constant, their shape is more variable than with the other images which would make it even more difficult to characterize this image by a single parameter such as fractal dimension. In general a single-valued index is likely to be inadequate for characterizing the texture of real images, and changes in an index over different scales need to be examined.

3.2. Trabecular bone images

The effect on the estimates of fractal dimension of correcting for noise and MTF effects in the bone images is shown in Table 3. The small standard deviations indicate the preciseness of the estimates and the small inter-subject variance. The noise added additional roughness to the images, and its removal results in a reduced estimate of the fractal dimensions. The reduction is small but measurable, and is not uniform. The smoother images (i.e. those measured at larger slice thicknesses) were affected most. Correction for the MTF of the CT scanner has a more significant effect on the power spectral density. Deconvolution of the scanner blurring results in higher estimates of the fractal dimension, which are independent of the particular scanner used. Once this correction is made, the estimates fall within the theoretically allowed range for a 2-dimensional image of between 2.0 and 3.0. On a log–log scale, division by (MTF) [2] corresponds to addition of an increasing amount as frequency increases: thus, this correction reduces the slope of the power spectrum by a constant amount and consequently increases the estimated fractal dimension by a fixed amount.

The corrected plots of radial power spectral density for the set of bone images from a typical vertebra are shown in Fig. 6. They are not linear over the complete range of spatial frequencies, indicating some multifractality. The fractal dimensions, estimated for mid-frequencies and averaged for the five vertebrae, decrease monotonically for larger acquisition slice thickness (Table 3) as expected. The fractal signatures of a typical set of bone images are shown in Fig. 7. Their ability to discriminate between the images is limited due to the small size of the images. Nevertheless, they show a clear trend with the fractal signature dropping with increasing slice thickness.

Fig. 8 shows the normalized lacunarity plots for a typical set of bone images. The lacunarity of the four images is similar for small neighbourhood sizes (up to about 3 pixels) and for large neighbourhood sizes (greater than about 20 pixels), but differs in the intervening range. Within this range, the lacunarity is largest for the 1 mm slice image and falls as the slice thickness is
Table 3
Parameters obtained from the Fourier power spectra of the bone images. (The estimates of fractal dimension ($D_a$, $D_b$, $D_c$) were obtained using spatial frequencies of 5–9 (cycles/image). Each estimate is the mean from five vertebrae. $\delta_{b-a}$ and $\delta_{c-b}$ are the differences in the estimates between $D_a$ and $D_b$ and $D_c$ and $D_b$ respectively.)

<table>
<thead>
<tr>
<th>Slice thickness (mm)</th>
<th>$D_a$ (uncorrected)</th>
<th>$D_b$ (corrected for noise)</th>
<th>$\delta_{b-a}$</th>
<th>$D_c$ (corrected for noise and MTF)</th>
<th>$\delta_{c-b}$</th>
<th>Mean width</th>
<th>rms width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.970±0.005</td>
<td>1.94±0.01</td>
<td>−0.028</td>
<td>2.76±0.02</td>
<td>0.816</td>
<td>3.83</td>
<td>4.72</td>
</tr>
<tr>
<td>3</td>
<td>1.945±0.005</td>
<td>1.91±0.01</td>
<td>−0.036</td>
<td>2.73±0.02</td>
<td>0.816</td>
<td>3.79</td>
<td>4.63</td>
</tr>
<tr>
<td>5</td>
<td>1.890±0.005</td>
<td>1.83±0.01</td>
<td>−0.058</td>
<td>2.65±0.02</td>
<td>0.817</td>
<td>3.82</td>
<td>4.68</td>
</tr>
<tr>
<td>10</td>
<td>1.840±0.006</td>
<td>1.76±0.01</td>
<td>−0.080</td>
<td>2.58±0.02</td>
<td>0.816</td>
<td>3.77</td>
<td>4.62</td>
</tr>
</tbody>
</table>

increased. This drop in lacunarity is the expected consequence of blurring, since blurring makes an image more homogeneous and, therefore, less lacunar. The blurring is due to partial volume averaging in the z-direction, and increases with increasing slice thickness. Analysis of the deviation of the lacunarity plots from linearity estimates the characteristic sizes (in pixels) of structure within the images to be 10.75±0.5, 10.30±0.4, 9.50±0.4 and 8.75±0.4 for the 1, 3, 5 and 10 mm slice images respectively. These differences are statistically significant at the $P<0.05$ level and confirm that important textural information is modified as slice thickness is increased between 1 and 10 mm due to z-direction partial voluming.

4. Discussion

Although macroradiographs of lumbar vertebrae [16], contact radiographs [23,39], and high-resolution CT [40] and MRI [41] images show a fractal nature to the trabecular bone, whether images of vertebral trabecular bone acquired at the much lower spatial resolution of CT are fractal over any significant range has been disputed. A preliminary study of CT images of vertebral trabecular bone concluded that the images were fractal and that osteoporotic patients could be distinguished from normal patients based on estimated fractal dimen-
sion [42]. However, a later study [43] using high-resolution nuclear magnetic resonance (NMR) images of vertebral bone found no fractal nature and suggested that the fractal properties of the trabecular network reported in the earlier CT study were artefactual and resulted from insufficient resolution and consequent partial voluming and noise. The box-counting algorithm used in the CT study was sharply criticized since it does not allow correction for additive system noise which otherwise falsely imparts a fractal nature. Furthermore, no attempt was made to correct for the limited resolution of the CT scanner, which affects the estimates of fractal dimension using this algorithm in a complicated way reducing the ability to discriminate between images with genuinely different fractal dimension.

Our study, using the power spectrum algorithm, was able to correct for noise and the effect of a limited MTF. Correcting for system noise results in a small, non-uniform reduction in the estimates of fractal dimension (Table 3). The results support those of a simulation study [11] in which noise was added to synthetic fractals. It showed that the effect on the estimated fractional dimension was negligible for signal-to-noise levels greater than about 40 dB, but quickly became significant for smaller signal-to-noise levels. Correcting for MTF of the CT scanner increases the estimated fractal dimensions by a larger, fixed amount (Table 3). Veenland et al. [11] using computer generated fractal images and an MTF modelled on the logit function, also reported a constant correction when the MTF was taken into account. Even after making these corrections, our results show that CT images of trabecular bone have only a limited fractal nature and can best be characterized by a fractal signature rather than a single fractal dimension (Fig. 7). The width of the radial power spectrum [whether characterized by the mean or RMS (Table 3)] was not a useful indicator of texture in CT images, with correlation coefficients of 0.493 and 0.487 with the corrected fractal dimension $D_0$, respectively, even though it has had some success as a texture measure in higher resolution images [33,34].

Lacunarity analysis has been successfully used in landscape ecology with optical [17] and synthetic aperture radar [18] images. Grey scale images were pre-processed to derive an ordered sequence of binary images based on the quartiles of the image histogram, and then each binary image was analysed separately. This treatment was successful because each binary image could be associated with scene features related to interactions with the illuminating radiation. The ‘dark’ pixels of the first quartile were produced by specular reflectors of microwaves [18] (e.g., bodies of open water) and by features that lack the spectral ‘red edge’ [17] (i.e., differential absorption of red and near infrared light) such as devegetated areas. Conversely, the ‘bright’ pixels of the fourth quartile were produced by dense tree canopies that exhibit both strong backscattering and a strong red edge. The middle half of the image histogram was dominated by speckle noise in the case of SAR [18] and mixed or boundary pixels in the case of optical data [17]. However, the pre-processing by quantiles is arbitrary and may introduce some bias; in most medical applications the objects of interest are well-defined and known a priori, so that a single threshold to produce two classes (foreground and background) is likely to be more appropriate. Thresholding a grey-scale image is equivalent to intersecting the surface with a horizontal plane. Threshold level has been reported to have a substantial effect on the estimated fractal dimension of trabecular bone images using the box-counting algorithm [44,45]. To minimize bias, we used the median value in each image as the threshold. For the binarized CT images the white pixels can be considered to represent trabecular bone and the black pixels marrow. The smooth range of intervening grey levels present in the original CT images are largely an artefact of the partial volume effect (due to the limited spatial resolution of the imaging system) rather than real intermediate values, so that binarizing these images does not significantly reduce their actual, as opposed to apparent, information content.

Even for images that are substantially fractal in nature perceptually quite different textures often produce similar estimates of fractal dimension [3,35], and fractal dimension is known to perform poorly in texture segmentation [46]. The fractal dimension of an image can be obtained from the slope of the Fourier power spectrum. Changing the contrast of an image does not change the slope; instead it scales the spectrum, shifting it up if the contrast is increased or down if the contrast is reduced. Thus, contrast and fractal dimension are independent metrics and both may be required to characterize texture adequately [47]. The fractal signature, computed in the frequency domain, is useful in discriminating texture at least amongst images that have similar contrast. Changing the contrast of an image has no effect on the resulting lacunarity plot, suggesting that lacunarity includes information on contrast [indeed, by definition (Eq. 3)], lacunarity is a normalized variance of pixel values over varying neighbourhood sizes]. Hence lacunarity by itself may be sufficient to characterize texture, resulting in a major advantage over fractal dimension [35,36].

Lacunarity is not predicated on self-similarity (fractality) but rather depends on the pattern of aggregation (viz. the relative distribution of bone and marrow in trabecular bone images), and is not sensitive to the boundaries of an image [37]. The lacunarity algorithm is simple to implement, depending only on local means and variances calculated for different window sizes throughout the image. Lacunarity plots can be generated in near real-time, following image acquisition and binarization. There is no need to correct for noise in the
image: speckle noise manifests as a spatially random texture and its contribution to lacunarity decays rapidly as window size increases. Lacunarity analysis gives readily interpretable graphic results, from which the size of a characteristic sub-structure and its degree of spatial organization can be obtained, and can be used with confidence even with small images of 32×32 pixels. It can distinguish between similar CT images without the need for correcting the images for scanner MTF (which depends on FOV and reconstruction algorithm). For CT images of trabecular bone (with a spatial resolution of about 1.1 mm), it suggests that a scan thickness of 1 mm or less is necessary if essential architectural information is not to be lost. The usual slice thickness for routine BMD estimation by CT imaging is 10 mm, to allow sufficient volume of bone to be sampled for an accurate estimate. Clearly, another image at 1 mm thick (or less) is required for textural characterization. This would result in a small additional dose to a patient. An image at an intermediate thickness is not a suitable compromise.

The characteristic size of about 10–11 pixels corresponds to an ordering or clumping of trabeculae into structural units of about 5 mm in size. Such a unit would comprise many trabeculae, whose individual size typically ranges from 0.1 to 0.3 mm with inter trabecular spaces varying from 0.2 to 2.0 mm [48]. The details of this structural organization of trabeculae may enable us to monitor the deteriorating bone quality associated with osteoporosis. High-resolution microcomputed tomography (μCT) systems [49,50] are now available to specifically reconstruct trabecular architecture within small tissue samples, and could be used to assess the effect of resolution on the estimates of fractal signature and lacunarity. We did not have access to such a system.

However, we have compared the texture of clinical CT images with high-resolution projection radiography images of vertebral trabecular bone [51] and validated the methodologies and their applicability to the clinical setting by showing that essential information on trabecular microarchitecture was identical.

The sensitivity of both the fractal signature and lacunarity techniques is evidenced by their ability to distinguish between CT images of vertebral trabecular bone that differed only in the slice thickness used in the tomographic scanning. Although our specimens were all of normal bone, this sensitivity suggests that the techniques may be useful in assessing the changes in trabecular architecture that accompany the progressive loss of bone density characteristic of osteoporosis and which are inadequately assessed by current bone mineral density (BMD) measures. We are currently studying a range of bone specimens to determine whether lacunarity analysis can characterize the trabeculation pattern with sufficient sensitivity to distinguish degrees of osteoporosis. In principle the lacunarity algorithm is applicable to grey-scale images [37,52], and thresholding the images should not be necessary: we are investigating the analysis and interpretation of such data.

Appendix A

Roughness or smoothness is an important feature of texture, and a classical means of measuring the smoothness of a function \( f(x) \) involves the Fourier transform. The Fourier transform of an absolutely integrable function \( f(x) \), denoted \( \hat{f}(\omega) \), is

\[
\hat{f}(\omega) = \int f(x)e^{-i\omega x}dx \tag{A1}
\]

There is a general relation between the smoothness of \( f(x) \) and the decay of \( |\hat{f}(\omega)| \) as \( |\omega| \to \infty \). For example, if \( \hat{f}(\omega) \) decays fast enough so that

\[
\int |\omega|^{2\beta} |f(\omega)|^2 d\omega < \infty \tag{A2}
\]

for \( \beta=0 \), then it can be shown that \( f(x) \) is square integrable. If Eq. (A2) holds for \( \beta=1 \), then \( f(x) \) is square integrable [and hence \( f(x) \) is continuous, but only just (i.e. rough)]. If we let \( \beta \) be the largest (supremum of all) \( \beta \) such that Eq. (A2) holds, then \( \beta \) can be interpreted as a smoothness index with 1 corresponding to rough and 3 corresponding to smooth for a 1-dimensional profile, or \( \beta=2 \) corresponding to rough and \( \beta=4 \) to smooth for a 2-dimensional function. Thus the radial Fourier power spectrum of rough (2-D) images will tend to fall as \( 1/\omega^2 \), showing a gradient of \(-2\) on a log–log plot; whilst smooth (2-D) images will fall as \( 1/\omega^4 \) with a gradient of \(-4\). At very low frequencies, corresponding to the bulk features of an object, the power spectrum may be fairly constant: and at very high frequencies, approaching the Nyquist frequency, system noise will dominate and the power spectrum will become constant again. This general shape has been reported in the power spectrum of MRI images of smooth body parts [53].

There are many different models used to describe fractals, but fractional Brownian motion (FBM) is the most useful mathematical model for the random fractals found in nature. It is an extension of the more familiar Brownian motion, which is the limiting case of a random walk. It has been shown that under restricted conditions (spatially isotropic surface, orthographic projection, Lambertian reflectance and uniform illumination) the fractal dimension of a (volume) image dictates the fractal dimension of the intensity surface of its projection, \( I(x) \). The intensity surface of the projected image, \( I(x) \), can be considered to be generated by fractional Brownian motion [11,47] if, for all \( x \) and \( \Delta x \),
where $F(p)$ is a cumulative distribution function of a random variable $p$ (typically a zero mean Gaussian, with variance $s^2$, i.e.

$$F(p) = \int_0^p \frac{1}{\sqrt{2\pi s^2}} \exp\left(-\frac{t^2}{2s^2}\right) dt$$

and $H \in [0, 1]$. Eq. (A3) shows that the change in the intensity divided by the scale at which the intensity measures are made (to the power $H$) has the same (Gaussian) probability distribution for all scales.

A consequence of Eq. (A3) is that increments of FBM have a Gaussian distribution with variance

$$<|l(x_2)-l(x_1)|^2> = |x_2-x_1|^{2H}s^2$$

where $H$, the Hurst parameter, has a value in the range [0,1] with $H=0$ corresponding to roughness and $H=1$ to smoothness. For $H=1/2$ this reduces to classical Brownian motion ($\Delta F=\Delta x$) with variance $s^2$.

FBM shows a statistical scaling behaviour. If $r>0$ is a scaling factor and $f$ is FBM with Hurst parameter $H$ and variance $s^2$, then so is $g(t) = r^{-H} f(rt)$. In other words, if the graph of $f$ is stretched out along the $t$ axis by a factor of $1/r$, and along the vertical axis by a factor of $r^{-H}$, then the resulting random process $g$ is statistically indistinguishable from $f$. Such non-uniform scaling is known as self-affinity (rather than self-similarity) and is more applicable to intensity images. It has been shown [8] that the scaling behaviour of a statistically self-affine fractal results in a fractal dimension, $D$, given by

$$D = E + 1 - H$$

where $E$ is the topological (Euclidean) dimension of the fractal (i.e. $E=2$ for a fractal surface).

Since there is a simple relationship between the Hurst parameter, $H$, and the magnitude of the slope of the Fourier power spectrum, $\beta$ viz:

$$\beta = 2H + E$$

then the fractal dimension of an image can be obtained from this slope using

$$D = 3E/2 + 1 - \beta/2$$

in general, or

$$D = 4 - \beta/2$$

in the case of a 2-dimensional image ($E=2$), where $D$ will be constrained to be between 2 (smooth) and 3 (rough).

The relationships between the various indices are shown in Fig. A1, for the case of an image with a topological dimension of two. Although $H$ was formally restricted to [0,1], its range may be extended by taking the derivative of the FBM with $H=0$ to give white (Gaussian) noise with $H=-1$ (corresponding to a slope, $\beta$, of zero and a fractal dimension, $D$, of 4). Thus the range of values for the fractal dimension of a 2-dimensional image normally extends from 2 (smooth) to 3 (rough), but can reach 4 (for white noise). [Using Eq. (A8), the fractal dimension of a 1-dimensional profile normally ranges from 1 (smooth) 2 (rough), but can reach 2.5 (for white noise)].

White noise can be used to synthesize a spatially isotropic fractional Brownian function (i.e., a fractal) by filtering it so that its power spectrum decays as $f^{-\beta}$ (or its radial power spectrum as $k^{-\beta/2}$), where $k=(u^2+v^2)^{1/2}$ and $u,v$ are the horizontal and vertical spatial frequencies). The transfer function of the required filter, $T(k)$, necessary to produce a fractional Brownian surface of fractal dimension $D$ is

$$T(k) \propto k^{-(4-D)}$$

so that to generate a sampled approximation to the fractal surface, the real and imaginary parts of each element of the discrete Fourier Transform of the white noise image should be multiplied by $(u^2+v^2)^{(4-D)/2}$. The inverse transform is then the required fractal. The test images of Section 2.1 were synthesized in this way.

References


