Homework 1 Solutions

1. Find the coefficients a, b, and c so that the graph of the polynomial $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

Solution Given the set of points we obtain a system of linear equations such that

$$\begin{aligned} 2 &= a + b + c \\ 6 &= a - b + c \\ 3 &= 4a + 2b + c \end{aligned}$$

Putting the system in matrix form we get

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 6 \\ 4 & 2 & 1 & 3 \end{array}\right).$$

Now we put our matrix into row echelon. We start by taking $R_1 - R_2$, secondly $4R_1 - R_3$, thirdly $R_2 - R_3$, and $\frac{1}{2}R_2$ to get

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & -9 \end{array}\right).$$

Finally doing the row operations $R_1 - R_2$, next do $R_3 + 3R_1$ and get

$$\left(\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & -9 \end{array}\right).$$

Dividing R_1 by 3, and R_3 by -3 we get our final answers to be $a = 1$, $b = -2$, and $c = 3$.

2. Find the coefficients a, b, c, and d so that the graph of the polynomial $f(x) = ax^3 + bx^2 + cx + d$ passes through the points $(-3, -2)$, $(-1, 2)$, $(1, 5)$ and $(2, 1)$.

Solution This problem had a typo. If it didn't, the method would be the same as the previous problem.

3. Find all solutions to the system

$$\begin{cases} (1-\mathrm{i})x_1 - \mathrm{i}x_2 = 0 \\ 2x_1 + (1-\mathrm{i})x_2 = 0 \end{cases}$$

where $\mathrm{i} \in \mathbb{C}$.

Solution. We do this by the elimination method. We multiply the first equation by 2 and the second equation by $1-\mathrm{i}$ and get

$$2(1-\mathrm{i})x_1 + 2\mathrm{i}x_2 = 0 \tag{1}$$

$$2(1-\mathrm{i})x_1 + (1-\mathrm{i})^2 x_2 = 0. \tag{2}$$

Subtracting (2) from (1) we get

$$2\mathrm{i}x_2 - (1-\mathrm{i})^2 x_2 = 0$$

Solve directly for x_2:

$$\begin{aligned} \left(2\mathrm{i} - (1-\mathrm{i})^2\right) x_2 &= 0 \\ 4\mathrm{i}x_2 &= 0 \\ x_2 &= 0. \end{aligned}$$

So if $x_2 = 0$ then this implies $x_1 = 0$.

4. The system

$$\begin{aligned} 2\sin\alpha - \cos\beta + 3\tan\gamma &= 3 \\ 4\sin\alpha + 2\cos\beta - 2\tan\gamma &= 10 \\ 6\sin\alpha - 3\cos\beta + \tan\gamma &= 9 \end{aligned}$$

is not linear, however, we can still use elimination if we are clever in regards to the trig functions. Does this system have a solution?

Solution. Let $a = \sin\alpha$, $b = \cos\beta$, and $c = \tan\gamma$. Then

$$2a - b + 3c = 3 \tag{3}$$

$$4a + 2b - 2c = 10 \tag{4}$$

$$6a - 3b + c = 9. \tag{5}$$

Multiply (3) by 3 and then subtract (5) and get

$$\begin{aligned} 8c &= 0 \\ c &= 0. \end{aligned}$$

If $c = 0$, then (3) and (4) become

$$\begin{aligned} 2a - b &= 3 \\ 2a + b &= 5. \end{aligned}$$

Take (3)+(4) and get

$$\begin{aligned} 4a &= 8 \\ a &= 2. \end{aligned}$$

This implies $b = 1$. But we are not done, we must put a, b, c back into our substitution:

$$\begin{aligned} 2 &= \sin\alpha \\ 1 &= \cos\beta \\ 0 &= \tan\gamma. \end{aligned}$$

We see that for α there is no solution. $\sin\alpha$'s range is $\{-1, 1\}$, therefore $\sin\alpha$ can never be 2. Even though solving for the other two we get $\beta = 2\pi n$ and $\gamma = \pi n$, where $n \in \mathbb{N}$, the system does not have a solution.