Many identities which we regard as algebraic have a geometric interpretation. The Pythagorean Theorem is an example of this. Most ancient mathematicians would have considered algebraic identities such as  
\[(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)\]
to be statements about the areas of rectangles.

To understand this, we have to change our point of view. Let \(x\) be a (positive) number. Instead of thinking of \(x\) as a number, think of it as a length of a line segment.

Now suppose we have two numbers \(x\) and \(y\). Individually \(x\) and \(y\) are lengths. Their sum \(x + y\) is the length of a line segment made up of two pieces, one of length \(x\) and one of length \(y\).

Suppose that \(x > y\) (so \(x - y > 0\)). What is their difference \(x - y\)? It is the length of the segment of length \(x\) with a segment of length \(y\) removed.

With this point of view, what is \(x^2\)? Well we say “\(x\) squared,” so it is not hard to see we are talking about the area of a square with sides of length \(x\).

What is the product of two number \(xy\)? It is the area of a rectangle with sides of length \(x\) and \(y\).

What about things like \(x^2 + xy\)? This is the sum of two areas, the square with sides \(x\) and the rectangle with sides \(x\) and \(y\).
Now let’s prove an identity. Algebraically it is easy to see that

\[ x(x + y) = x^2 + xy. \]

Can we show this geometrically? Yes, we just have to make geometric sense of both sides of the equation. We already know what the Right Hand Side represents. What about the Left Hand Side? Well, it is a product and products are areas of rectangles. Which rectangle? The rectangle with sides of length \( x \) and \( (x + y) \).

\[ x(x + y) = x \]

\[ = x \]

\[ = x = x^2 + xy \]

We can divide the rectangle into two rectangles as shown above. The sum of the areas of the two rectangles is equal to the area of the first rectangle, and we have show our identity.

Similarly, we can show that

\[ x^2 - xy = x(x - y). \]

The Right Hand Side is just a rectangle with sides of length \( x \) and \( x - y \). The Left Hand Side is the square with sides of length \( x \) with a rectangle (sides \( x \) and \( y \)) removed.

\[ x^2 - xy = x \]

\[ = x \]

\[ = x = x(x - y) \]

There is a larger point here. Algebra and Geometry are very closely related in the early history of Mathematics. This is also the case in early Greek mathematics. Later, especially after Euclid (c. 300 BC), Algebra and Geometry become separate subjects.