Final Review Sheet part 1

This is a collection of exercises which are based upon the material we covered in class. This should not be taken as an implication that these problems look like those which will be on the final. The concepts they embody, however, are important to know for the final.

This weekend I'll put up a second set of review problems with some solutions.

NOTE: This review is an augmentation of the midterm 1 and midterm 2 reviews. When reviewing for the final, please look at those as well as this one.

Some ideas which are important to know (chapters 9-11, 13–14 for chapters 7–8 see midterm 1 review):

- Sets: intersection, union, interval notation
- Inequalities: Absolute value, one variable inequalities, two variable inequalities, graphing inequalities in one and two variables
- Radical expressions: radicals and exponents, simplifying radical expressions, multiplying, dividing, adding, and subtracting radical expressions, solving radical equations
- Issues involving squares and square roots: Pythagorean theorem, 30-60-90 and 45-45-90 triangles, working with complex numbers
- Quadratic equations: factoring, completing the square, the quadratic formula, applications of quadratic equations, equations and expressions reducible to quadratics
- Quadratic functions: graphs, roots, x and y intercepts, the location of the vertex, shifting and stretching the graphs
- Sequences and series: arithmetic and geometric sequences, finding the $n^{th}$ term of a sequence, $\sum$ notation, finding the partial sum of an arithmetic/geometric series, finding the sum of an infinite geometric series
- Conic sections: circles, finding the center and radius of a circle from an equation

1. Simplify the expression
   \[
   \frac{\sqrt{16x^2y}}{\sqrt{x^5y}}
   \]

2. For the following quadratic function, find the x-intercepts (if they exist), the y-intercept, the location of the vertex, and use this information to sketch a graph the function.
   \[
   y = 2x^2 - x - 6
   \]

3. In a right triangle, the hypotenuse has length 13, and one of the legs has length 12. What is the length of the remaining side?

4. Perform the operations and simplify. Express all variables with positive exponents.
   (a) $\sqrt{20x^2y} \cdot \sqrt{5xy}$
   (b) $\frac{3x^3y^{-2}}{x^2}$
(c) $\sqrt{20} + \sqrt{45}$

5. Rationalize the denominators
   
   (a) $\frac{3}{\sqrt{29}}$
   
   (b) $\frac{x}{\sqrt{y}}$
   
   (c) $\frac{1 + \sqrt{2}}{3 - \sqrt{3}}$

6. Find the slope, x-intercept, and y-intercept of the line
   
   $5x - 3y = 2,$
   
   and sketch its graph.

7. Graph the solution to the inequality $|x - 3| \leq 4$.

8. Graph the solution to this system of inequalities
   
   $\begin{align*}
   -2x + 3y & \geq -9 \\
   x + y & \leq 2 \\
   x & \geq -1
   \end{align*}$

9. Perform the operations
   
   a. $(3 - i) + (-6 + i)$
   
   b. $(-4 + 3i) - (5 + i)$
   
   c. $(3 - 2i)(-2 + i)$
   
   d. $\frac{3 - i}{1 + 3i}$

10. Solve the equation $2z^2 - 5z - 8 = 0$.

11. Find the center and radius of the circle $x^2 + y^2 + 10x - 4y = 0$.

12. The height at time $t$ for a ball thrown in the air is $h(t) = 50 + 40t - 5t^2$ meters. When will the ball reach its maximum height, and what is that height?

13. A circle has center $(-4, 1)$ and radius 5. Is the origin on the circle?

14. Solve the equation
   
   $t - \sqrt{t} - 2 = 0$

15. Find the common difference and the 653rd term of the arithmetic sequence
   
   $-64, -57, -50, -43, -36, \ldots$

16. Find the sum of this arithmetic series
   
   $1 + 2 + 3 + 4 + \cdots + 198 + 199 + 200.$

17. Find the sum of the geometric series
   
   $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$