1. \[ \sqrt[3]{16x^2y} = \sqrt[3]{2^4x^2} \cdot y = 2 \sqrt[3]{2^2x^2} \cdot y = \frac{2\sqrt[3]{2}}{x} \]

2. The \( x \)-intercepts occur where \( f(x) = 0 \). So \( 2x^2 - x - 6 = 0 \). Factoring, we get \( (2x+3)(x-2) = 0 \), which gives \( x = -\frac{3}{2} \) or \( x = 2 \).

The \( y \)-intercepts occur where \( x = 0 \), so \( f(0) = -6 \); therefore, the \( y \)-intercept is -6.

The \( x \)-coordinate of the vertex is given by the vertex formula; \( x = -\frac{-1}{2a} = \frac{1}{4} \). We plug this in \( f(x) \) to get the \( y \)-coordinate:

\[ f \left( \frac{1}{4} \right) = 2 \left( \frac{1}{4} \right)^2 - \left( \frac{1}{4} \right) - 6 = -\frac{1}{8} - 6 = -\frac{49}{8} \]

The graph can be made by plotting these points and drawing the parabola through them, keeping in mind where the vertex is as well as that the parabola open upward.

3. Since we have a right triangle, we use the Pythagorean theorem.

\[ 12^2 + b^2 = 13^2 \]
\[ 144 + b^2 = 169 \]
\[ b^2 = 25 \]
\[ b = 5 \]

So the other side has a length of 5. (Note: we take the positive value for \( b \) since lengths aren’t negative.)

4. Perform the operations and simplify. Express all variables with positive exponents.

(a) \[ \sqrt[4]{20x^3y} \cdot \sqrt[4]{5xy} = \sqrt[4]{20x^3y \cdot 5xy} = \sqrt[4]{100x^4y^2} = 10x^2y. \]

(b) \[ \frac{3x^2y^2}{3x^2y^2} = \frac{3x^2y^2}{3x^2y^2} = \frac{3x^2y^2}{x^2y^2} = \frac{9x^2}{x^2} = 9y \]

(c) \[ \sqrt[4]{20} + \sqrt[4]{45} = \sqrt[4]{4 \cdot 5} + \sqrt[4]{9 \cdot 5} = 2\sqrt[4]{5} + 3\sqrt[4]{5} = 5\sqrt[4]{5} \]

5. Rationalize the denominators

(a) \[ \frac{3}{\sqrt{2y}} = \frac{3 \cdot \sqrt{2y}}{\sqrt{2y} \cdot \sqrt{2y}} = \frac{3\sqrt{2y}}{2y} \]

(b) \[ \frac{x}{\sqrt{y}} = \frac{x \cdot \sqrt{y} \cdot \sqrt{y}}{\sqrt{y} \cdot \sqrt{y}} = \frac{x \sqrt{y^2}}{y} \]

(c) \[ \frac{1+\sqrt[3]{2}}{3-\sqrt[3]{3}} = \frac{1+\sqrt[3]{2} \cdot 3+\sqrt[3]{3}}{3-\sqrt[3]{3}} = 3+\sqrt[3]{2} = \frac{3+\sqrt[3]{2} + 3\sqrt[3]{2} + \sqrt[3]{6}}{6} = \frac{3+\sqrt[3]{2} + 3\sqrt[3]{2} + \sqrt[3]{6}}{6} \]

6. Solving for \( y \), we get

\[ y = \frac{5}{3}x - \frac{2}{3}. \]

The slope is \( m = \frac{5}{3} \), the \( y \)-intercept is \(-\frac{2}{3}\). We set \( y = 0 \) to get the \( x \)-intercept: \( 0 = \frac{5}{3}x - \frac{2}{3} \) gives \( x = \frac{2}{5} \).

The graph follows by drawing a line through these points.
7. Splitting the absolute value we must have \( x - 3 \leq 4 \) and \(-(x - 3)\leq 4\). We solve this to get \( x \leq 7 \) and \( x \geq -1 \). The graph is just the graph on the number line of the interval \([-1, 7]\).

Note: We are using the conjunction *and* since both inequalities must be true. If instead we had that the absolute value was *greater than* or equal to a number, then we’d use the conjunction *or*.

8. Graph the solution to this system of inequalities

\[
\begin{align*}
-2x + 3y &\geq -9 \\
x + y &\leq 2 \\
x &\geq -1
\end{align*}
\]

Solving for \( y \) where we can, we get the system of inequalities

\[
\begin{align*}
y &\geq \frac{2}{3}x - 3 \\
y &\leq -x + 2 \\
x &\geq -1
\end{align*}
\]

The graph of this is the region above the line \( y = \frac{2}{3}x - 3 \), below the line \( y = -x + 2 \), and to the right of the line \( x = -1 \). This region forms a triangle in the plane with corners \((-1, 3)\), \((-1, \frac{-11}{3})\), and \((3, -1)\). Since all the inequalities involved are \( \leq \) or \( \geq \), the region includes the entire boundary of the triangle, too.

9. Perform the operations

a. \((3 - i) + (-6 + i) = -3\)
b. \((-4 + 3i) - (5 + i) = -9 + 2i\)
c. \((3 - 2i)(-2 + i) = -6 + 3i + 4i - 2i^2 = -6 + 7i - 2(-1) = -4 + 7i\)
d. \(\frac{3 - i}{1 + 3i} = \frac{3 - i}{1 + 3i} \cdot \frac{1 - 3i}{1 - 3i} = \frac{3 - 9i - i + 3i^2}{1 - 3i + 3i - 9i^2} = \frac{-10i}{10} = -i\)

10. Solve the equation \(2z^2 - 5z - 8 = 0\).

\[
z = \frac{-((-5)\pm \sqrt{(-5)^2 - 4(2)(-8)}}{2(2)} = \frac{5 \pm \sqrt{25 + 64}}{4} = \frac{5 \pm \sqrt{89}}{4}
\]

11. Find the center and radius of the circle \(x^2 + y^2 + 10x - 4y = 0\).

We complete the square:

\[
(x^2 + 10x) + (y^2 - 4y) = 0 \\
(x^2 + 10x + 25) + (y^2 - 4y + 4) = 0 + 25 + 4 \\
(x + 5)^2 + (y - 2)^2 = (\sqrt{29})^2
\]

Thus the center is at \((-5, 2)\), and the radius is \(\sqrt{29}\).
12. The height at time $t$ for a ball thrown in the air is $h(t) = 50 + 40t - 5t^2$ meters. When will the ball reach its maximum height, and what is that height? Since the coefficient of $t^2$ is negative, this quadratic function describes a parabola which points down. Thus it has a maximum which is located at the vertex. So we find the location of the vertex $t = -\frac{40}{2(-5)} = 4$. So the maximum occurs at $t = 4$, and we plug this in to find the maximum value:

$$h(4) = 50 + 40(4) - 5(4)^2 = 130.$$ 

13. A circle has center $(-4,1)$ and radius 5. Is the origin on the circle? The equation for this circle is $(x - (-4))^2 + (y - 1)^2 = 5^2$. The origin has coordinates $(0,0)$, so we plug in this $(x,y)$ value to the equation.

$$(x + 4)^2 + (0 - 1)^2 = 25$$
$$4^2 + (-1)^2 = 25$$
$$17 = 25$$

Since the point $(0,0)$ does not satisfy this equation, we know that it is not on the circle.

14. Solve the equation

$$t - \sqrt{t} - 2 = 0$$

Notice here that we have a $(\sqrt{t})^2$ term, a $\sqrt{t}$ term, and a constant term. So if we let $w = \sqrt{t}$, then we get the equation

$$w^2 - w - 2 = 0,$$

which we then factor to get $w = 2$ or $w = -1$. Since we are solving for $t$, we plug it back in to get $\sqrt{t} = 2$ or $\sqrt{t} = -1$. Since the square root function doesn’t give negative numbers, we eliminate the second possibility to get only $\sqrt{t} = 2$, and then $t = 4$. If we plug this in to check, it works.

15. Find the common difference and the 653rd term of the arithmetic sequence

$$-64, -57, -50, -43, -36, \ldots$$

The first term is $a_1 = -64$, and the common difference is $d = 7$. We are given $n = 653$. Using the term of an arithmetic sequence formula, we get $a_{653} = -64 + 7(653 - 1) = 4500$.

16. Find the sum of this arithmetic series

$$1 + 2 + 3 + 4 + \cdots + 198 + 199 + 200.$$ 

We see $a_1 = 1$, $n = 200$, and $a_{200} = 200$. Therefore $S = \frac{200}{2}(1 + 200) = 20100$.

17. Find the sum of the geometric series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

The first term is $\frac{1}{2}$, and the second is $\frac{1}{4}$, so the common ratio is $r = \frac{1}{2}$. The first term is $a_1 = \frac{1}{2}$. Then the sum is

$$S = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{1}{2 - 1} = 1$$