The midterm will cover the sections 3.1-3.6.
This review is designed to be an aid in study for the midterm. It is not
designed to mimic exactly what will be on the exam. The problems on the
exam may be different than those in this document. The ideas, however, which
are used here will be of great use on the midterm.
Here are some useful general things to know:

• Limits, computing limits, limit properties (p.140), estimating limits from
graphs (such as exercises 3.1.25-28), limit of a difference quotient

• Continuous functions, definition of continuity, identifying continuous func-
tions and discontinuities from graphs, continuity properties (p.153), asympt-
totes and continuity, drawing a graph with certain limit and continuity
properties

• Limit properties of functions

• Continuity

• Average rate of change

• Definition of derivative and finding the derivative of a function from the
definition

• Instantaneous rate of change

• Derivatives of various forms of functions:
  – sums and differences
    \[(f + g)' = f' + g'\]
    \[(f - g)' = f' - g'\]
  – constants
    \[\frac{d}{dx}(c) = 0\]
  – The power rule
    \[\frac{d}{dx}(x^n) = nx^{n-1}\]
  – a constant times a function
    \[(c \cdot f)' = c \cdot f'\]
  – the product rule
    \[(fg)' = f'g + fg'\]
the quotient rule
\[ \left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2} \]

chain rule: power form
\[ (f^n)' = nf^{n-1} \cdot f' \]

Finding the tangent line to a function at a point.

Marginal cost

Here are some exercises to help in your studies.

1. Using the properties of limits, find
\[ \lim_{x \to 3} \sqrt{4x + 4}. \]

2. Consider the function
\[ f(x) = \frac{x^2 - 9}{x - 3}. \]
   (a) Using the properties of limits, find \( \lim_{x \to 3} f(x) \), if it exists.
   (b) Find \( f(3) \), if it exists.
   (c) Does \( \lim_{x \to 3} f(x) = f(3) ? \) If not, can you redefine \( f(3) \) so that it does?

3. Consider the graph of the function on p.161.
   (a) For which \( x \) values in the domain is this function discontinuous?
   (b) For these \( x \) values you found, for which ones can you define or redefine
       \( f(x) \) at that point so that the function is continuous.
   (c) For which points \( x = c \) will the function always be discontinuous,
       regardless of the value of \( f(c) ? \)

4. Sketch a graph of a function \( y = g(x) \) which has these properties:
   (a) \( \lim_{x \to 2^-} g(x) = 1 \)
   (b) \( \lim_{x \to 2^+} g(x) = 3 \).
   (c) \( g(2) = -1 \).
   (d) \( g(x) \) is continuous for all \( x \neq 2 \).
   (e) \( g(-2) = 4 \).
   (Note: There are a lot of possible solutions to this question.)

5. Let \( g(x) = 2x^2 - 4x + 7 \). What is the average rate of change of \( g(x) \)
   between \( x = -1 \) and \( x = 4 \)?
6. Let \( P(t) = t^2 - 10t - 30 \) be the profit made by Leon’s Lemonade Stand on day \( t \) of operation.

   (a) What is the average profit per day between days 10 and 20?
   (b) What is the average profit per day between days 10 and \( 10 + h \)?
   (c) What is the instantaneous profit per day at day 10?

7. Let \( f(x) \) be a function. What is the definition of the derivative of \( f \)?

8. **From the definition**, find the derivative of the function \( g(x) = 3x^2 - 5x + 2 \).

9. Let \( r(x) = \frac{x+1}{\sqrt{x}} \). What is the instantaneous rate of change of \( r(x) \) when \( x = 2 \)?

10. Find the derivative.
    \[
    f(x) = 3x^5 - 193x^2 + 345 - \frac{1}{\sqrt{x}}
    \]

11. Find the derivative.
    \[
    g(x) = (-3x^2 + 5x + 13)(x^4 - 3)
    \]

12. Find the derivative.
    \[
    r(x) = \frac{3x^2 - 7x + 2}{x^2 - x + 3}
    \]

13. Find the derivative.
    \[
    s(x) = (2x^2 + 5x + 30)^{10}
    \]

14. Find the derivative.
    \[
    m(x) = 476993875629863596294658963924865928639723
    \]

15. Find the derivative.
    \[
    n(x) = \frac{(3x + 1)^6}{(2x - 5)^4}
    \]

16. Let \( r(x) = x^{1500} - 600x^3 + 3065476346 \). Find the numerical value of \( r'(-1) \), and simplify it.

17. Find the tangent line to the function \( g(x) = 3x^2 - 5 + \frac{1}{x} \) at the point \( x = 2 \).

18. (See p.187-189) Let \( C(x) = 50000 + 2000x + x^2 \) be the cost of manufacturing \( x \) scooters.
   (a) How much does it cost to produce 100 scooters?
(b) What is the marginal cost at a production level of 100 scooters?
(c) Using the marginal cost, how much would it approximately cost to produce the 101st scooter?
(d) What is the exact cost of producing the 101st scooter?

19. Let $I(t)$ be the income of a company in year $t$. Interpret what it means for
   \[ I'(2008) = 260000. \]

20. Let $P(t)$ be the number of hula hoops owned by children in year $t$. Suppose you know that $P(2005) = 100000$ and $P'(2005) = 5000$. Based on the data from the year 2005, how many hula hoops would you estimate children will own in the year 2010?