Math 150 Midterm 1 Review
Midterm 1 - Monday Sept. 21 or Tuesday Sept. 22

The midterm will cover the sections we covered 1.1-2.7.
This review is designed to be an aid in study for the midterm. It is not
designed to mimic exactly what will be on the exam. The problems on the
exam may be different than those in this document. The ideas, however, which
are used in the solutions here will be of great use on the midterm.

Here are some useful general things to know:
• Definition of function, domain, range
• Graphs of functions, vertical line test, shifting and reflecting graphs, piecewise-
  defined functions
• Combining functions: via algebraic operations and via composition
• The inverse of a function
• Exponents and logarithms
• Limits (and asymptotes), calculating limits
• Continuity, left and right handed continuity, removable discontinuities, the
  IVT
• Average rate of change
• The definition of the derivative, the instantaneous rate of change,
  instantaneous velocity, the slope and equation of a tangent line

Here are some problems to help in your studies. Note that when you solve
the problems, it is ok to leave roots, exponents and logarithms in the answer
without evaluating a numerical answer when the calculation is out of the realm
of quick hand calculation, e.g. $5^{30}$ or $\log_{3} 23$. However, you should evaluate
relatively simple things like $3^2$, $5^{-1}$, $\log_3 1$, $\ln e$, $\sqrt{25}$, and so on.

When solving a problem, always justify your answer unless the problem says
just state something. It is important to be able to support your conclusion (i.e.
this is where partial credit comes in).

1. Given the graph of the function $f(x) = |x|$, sketch the graph of $y =
   -f(x - 2) + 1$.

2. Sketch the graph of

   $$g(x) = \begin{cases} 
   3, & x < -2 \\
   x^2, & -2 \leq x \leq 1 \\
   3 - x, & x > 1 
   \end{cases}$$
3. Given the quadratic equation

\[ h(x) = x^2 - 6x + 10, \]

(a) find the \( x \)-intercepts (if they exist),
(b) the \( y \)-intercept,
(c) the coordinates of the vertex,
(d) and sketch the graph of this function.
(e) Without actually finding the \( x \)-intercepts, how can you find out if there are 0, 1, or 2 intercepts?

4. What is the most number of roots that this polynomial can have? Why?

\[ p(x) = 3x^7 + \pi x^{32} - x^2 + 17022x^{1001} \]

5. Consider the function

\[ r(x) = \frac{2x - 3}{x + 1}. \]

(a) What are the \( x \)-intercept(s)?
(b) What are the \( y \)-intercept(s)?
(c) Find the horizontal and vertical asymptotes.
(d) Using your information, sketch a graph of this function.

6. Solve the equation

\[ 3^{5x+5} = 9^{3x+1}. \]

7. Solve the equation

\[ 4^{x^2} \cdot 64 = 4^{4x}. \]

8. Evaluate the following expressions

(a) \[ \log_3 9 \]
(b) \[ \log_4 32 \]
(c) \[ \log_5 5^29 \]
(d) \[ 6^{\log_6 7} \]

9. Solve the equations

(a) \[ 4^{3x-1} = 50 \]
(b) \[ 3^{x+1} = 5^x \]

(c) \[ \log_5(x - 1) + \log_5(x + 3) = 1 \]

10. Let \( f(x) = 3x^2 - 5x + 2 \) and \( g(x) = -2x + 1 \). Find the following combinations:
   
   (a) \( f(g(f(2))) \)
   (b) \( f(x) + 7g(x) \)
   (c) \( f(x) \cdot g(x) \)
   (d) \( f(g(x)) \)
   (e) \( g(f(x)) \)
   (f) \( f(f(x)) \)
   (g) \( g(g(g(x))) \)

11. In section 1.6, in problems 3–8, which of the functions are one-to-one?

12. Find the inverse function of \( h(x) = -\sqrt{x + 2} + 1 \). Be sure to indicate the domain of \( h^{-1}(x) \).

13. Compute the limits, or state they why do not exist:
   
   (a) \[ \lim_{t \to 2} 7t^3 - t^2 + 3t - 1 + \frac{1}{t} \]
   
   (b) \( \lim_{x \to 4} \frac{1}{(x - 4)^3} \)
   
   (c) \( \lim_{x \to 4} \frac{1}{(x - 4)^3} \)
   
   (d) \( \lim_{x \to 4} \frac{1}{(x - 4)^2} \)
   
   (e) \( \lim_{z \to 4} \frac{z^2 - 10z + 24}{2x^2 - 7x - 4} \)
   
   (f) \( \lim_{x \to \infty} \frac{x^2 - 9}{x + 3} \)
14. Let \( f(x) = \frac{3x^2 + 1}{x^2 + 6x + 19} \).

(a) Compute the limit:
\[
\lim_{x \to \infty} f(x),
\]
and call this limit \( L \).

(b) Find a number \( M \) on the positive real axis so that whenever \( x \geq M \),
\[ |f(x) - L| < \frac{1}{1000}. \] (Note: This is from the definition of a limit to
infinity. If we go past some point \( M \) then we’re within \( \frac{1}{1000} \) of the
limit. That the limit exists means that we could replace the
\( \frac{1}{1000} \) with any number, no matter how small, and that we always could
find a suitable \( M \).

15. For the following functions, state where they are continuous (you may use
interval notation or \( a < x < b \) notation, or equivalent; here we’re not
limited to only what Webwork recognizes)

(a) \( f(x) = x^2 - 3x + 2 \)
(b) \( g(x) = \frac{x^3}{x^3 - 1} \)
(c) \( h(x) = \frac{x^2 - 3x + 2}{x - 1} \)
(d) \( r(x) = \ln x \)
(e) \( s(x) = e^{x^3} - 300x^{50} - 70023 \)
(f) \( p(x) = \begin{cases} 2x, & x \geq 3 \\ x^2, & x < 3 \end{cases} \)

16. Problem 2.5.3 in Stewart. Also state whether the discontinuities of \( f(x) \)
are removable.

17. For what value(s) of \( c \) is this function continuous? There may be no real
solution to this.
\[
f(x) = \begin{cases} cx^2 + 3x + c^2, & x \leq 1 \\ -4cx + 9, & x > 1 \end{cases}
\]

18. The function
\[ p(z) = z^3 + z^2 - 2z - 29 \]
has exactly one root in the interval \([-4, 4]\). Find which of the intervals
\([-4, -3], [-3, -2], [-2, -1], \ldots, [3, 4]\) contains it. Justify your answer.
19. **Let \( f(x) \) be a function. State the definition of the derivative of \( f(x) \).**

20. **Without computing the derivative, state what the derivative of \( f(x) = -5x + 2 \) is. Now compute it. Is it what you expect?**

21. **Let \( g(t) = \frac{1}{t} \).**
   
   (a) What is the average rate of change of \( g(t) \) between \( t = 3 \) and \( t = 5 \)?
   
   (b) What is the average rate of change of \( g(t) \) between \( t = 3 \) and \( t = 3+h \)?
   
   (c) What is the instantaneous rate of change of \( g(t) \) at \( t = 3 \)?

22. **Let \( f(x) = x^2 - 13x + 7 \).**
   
   (a) What is the slope of the tangent line to \( y = f(x) \) when \( x = 2 \)?
   
   (b) Find the equation of the tangent line to \( y = f(x) \) when \( x = 2 \).
   
   (c) What is the slope of the tangent line to \( y = f(x) \) when \( x = c \)?

23. **A particle moves on the \( x \)-axis, and its position at time \( t \) seconds is given by**
   
   \[ x(t) = t^3 - 2t^2 \text{ meters}. \]
   
   (a) What is its velocity at time \( t = -1 \)? Include the units.
   
   (b) Is it traveling in the positive direction or negative direction at this time?