MATH 140 MIDTERM 1
WEDNESDAY MARCH 5

There are 9 questions on this exam, and you will have 70 minutes to take it. No calculators or notes are allowed, and the only scratch paper you may use is the paper provided. Each question is weighted the same in computing the grade.

If you are having difficulty with a problem, it may help to move on to another problem and come back later.

Be sure to show all of your work.

Name: ____________________________
1. Consider the function $f(x)$ given by this graph:

(a) Sketch a graph of $y = f(x - 3) + 2$.

(b) Sketch a graph of $y = -f(-x)$.
2. Find the equation of the line through the points \((1, 2)\) and \((9, 7)\).

\[
\begin{align*}
\text{m} &= \frac{7-2}{9-1} = \frac{5}{8} \\
\text{D} \quad y-2 &= \frac{5}{8}(x-1) \\
y-2 &= \frac{5}{8}x - \frac{5}{8}
\end{align*}
\]

\[
\begin{align*}
y &= \frac{5}{8}x - \frac{5}{8} + 2 \\
y &= \frac{5}{8}x + \frac{11}{8}
\end{align*}
\]

3. You are managing a bookstore and are having a promotional sale. Let \(g(x)\) be the number of books sold on day \(x\) of the promotion. Suppose you sell 300 books on day 3 and 500 books on day 7.

(a) Assuming \(g(x)\) is a linear function, find the equation for \(g(x)\).

\[
\begin{align*}
g(3) = 300 & \implies (3, 300) \\
g(7) = 500 & \implies (7, 500) \\
m &= \frac{500-300}{7-3} = \frac{200}{4} = 50 \text{ books/day} \\
y-300 &= 50(x-3) \\
y &= 50x + 150
\end{align*}
\]

(b) Using your function you found in the first part, how many books will be sold on day 15 of the promotion?

\[
g(15) = 50 \cdot 15 + 150 = 750 + 150 = 900
\]
4. Consider the quadratic function \( f(x) \) and its graph \( y = f(x) \):

\[
f(x) = x^2 + 6x + 5
\]

(a) What is the \((x, y)\)-coordinate of the vertex?

\[
f(x) = x^2 + 6x + 5
= x^2 + 6x + 9 + 5 - 9
= (x + 3)^2 - 4
\]

\[
\text{vertex is at } (-3, -4)
\]

(b) What are the \(x\)-intercepts, if they exist?

\[
0 = x^2 + 6x + 5
\]

\[
0 = (x + 5)(x + 1)
\]

\[
x = -5 \quad \text{and} \quad x = -1
\]

(c) What is the \(y\)-intercept, if it exists?

\[
f(0) = 0^2 + 6 \cdot 0 + 5 = 5
\]

\[
(0, 5)
\]

(d) Using the information you found above, sketch a graph of \( y = f(x) \).

To get \( y = (x + 3)^2 - 4 \),

we start with \( y = x^2 \)

and shift 3 to the left

and 4 down.

We also have the

points from the previous

parts.
5. Solve the equation \(5^{2x-1} = 43\).

\[
\ln 5^{2x-1} = \ln 43
\]

\[
(2x-1) \ln 5 = \ln 43
\]

\[
2x \ln 5 - \ln 5 = \ln 43
\]

\[
2x \ln 5 = \ln 43 + \ln 5
\]

\[
x = \frac{\ln 43 + \ln 5}{2 \ln 5}
\]

Using logs with other bases also works:

\[
\log_5 5^{2x-1} = \log_5 43
\]

\[
2x-1 = \log_5 43
\]

\[
2x = \log_5 43 + 1
\]

\[
x = \frac{1 + \log_5 43}{2}
\]

6. You invest $3000 in an account paying 10% interest, compounded continuously. How long will it take the account to be worth $15000?

\[
P e^{rt} = A
\]

\[
15000 = 3000 e^{.1t}
\]

\[
\frac{15000}{3000} = 3000
\]

\[
5 = e^{.1t}
\]

\[
\ln 5 = \ln e^{.1t}
\]

\[
\ln 5 = (.1t)
\]

\[
\ln 5 = (-10 \ln 5)
\]

Using logarithms with other bases works, but since the base of the exponent is \(e\), \(\ln\) works the best for this situation.
7. Combine the following expressions into a single logarithm such that no negative exponents appear and that each variable appears at most once.

(a)
\[7 \log_b x - 3 \log_b y + \log_b \left( \frac{1}{z} \right)\]
\[\log_b x^7 + \log_b y^{-3} + \log_b \left( \frac{1}{z} \right)\]
\[\log_b \left( \frac{x^7 y^{-3}}{z} \right)\]

(b)
\[2 \log_3 a + 4 \log_3 b^{-1} + 3 \log_3 c\]
\[\log_3 a^2 + \log_3 b^{-4} + \log_3 c^3\]
\[\log_3 \left( \frac{a^2 b^{-4} c^3}{b^4} \right)\]

8. Solve this equation.
\[\log_3 x + \log_3 (x + 8) = 2\]
\[\log_3 [(x)(x + 8)] = 2\]
\[\log_3 (x(x + 8)) = 2\]
\[3 = 3^2\]
\[x(x + 8) = 9\]
\[x^2 + 8x - 9 = 0\]
\[(x + 9)(x - 1) = 0\]
\[x = -9 \text{ or } x = 1\]

Checking:
\[x = -9:\]
\[\log_3 (-9) + \log_3 (-9 + 8) = ?\]
\[\times \text{ logs of negative numbers}\]
\[x = 1:\]
\[\log_3 1 + \log_3 (1 + 8) = 2\]
\[0 + \log_3 9 = 2\]
\[2 = 2 \checkmark\]
\[x = 1\]
9. Consider the graph of this function, and answer the following questions:

(a) Find \( \lim_{{x \to -1}} f(x) = \frac{-2}{3} \)

(b) Find \( f(-1) = \frac{3}{2} \)

(c) Find \( \lim_{{x \to 2^+}} f(x) = \frac{3}{2} \)

(d) Find \( \lim_{{x \to 2^-}} f(x) = \frac{3}{2} \)

(e) Find \( f(2) = \frac{3}{2} \)

(f) For which \( x \)-values is the graph of \( y = f(x) \) discontinuous? \( x = -1, x = 2, x = 3 \)

(g) For which of the \( x \)-values where the graph is discontinuous can you redefine \( f(x) \) so that it is continuous there? Justify your answer. \( x = -1, \) and \( x = 3 \)

At \( x = -1 \), if we define \( f(-1) = \frac{3}{2} \), then the limits match with the function value at \( x = -1 \), and the function would be continuous there.

At \( x = 3 \), the limit exists, so if we let \( f(3) = \frac{3}{2} \), then by the same reasoning above, it will be continuous.

At \( x = 2 \), however, the limit doesn't exist, so we can never make the limit & function value match, no matter what value of \( f(2) \) we choose.