Math 140 Midterm 1 Review
Midterm 1 - Wednesday March 5

The midterm will cover the sections we covered 1.1-3.2. This review is designed to be an aid in study for the midterm. It is not designed to mimic exactly what will be on the exam. The problems on the exam may be different than those in this document. The ideas, however, which are used in the solutions here will be of great use on the midterm.

Here are some useful general things to know:

- Definition of function, domain, range
- Graphs of functions, vertical line test, shifting and reflecting graphs, piecewise-defined functions
- Linear functions, slope, intercepts, finding the equation of a line (point-slope formula, etc.)
- Quadratic functions, intercepts, location of vertex, graphing a quadratic function
- Polynomials, degree, roots, turning points
- Rational functions, intercepts, asymptotes, sketching a graph
- Exponentials, properties of exponentials (p.98-99 and others we discussed in class), exponential equations, graphs of exponential functions, 1-1 functions (and horizontal line test), $e^x$, compound interest and population growth
- Logarithms, rewriting exponentials, properties of logarithms (p.114-115 and others we discussed in class), logarithmic expressions, solving exponential and logarithmic equations
- Limits, computing limits, limit properties (p.140), estimating limits from graphs (such as exercises 3.1.25-28), limit of a difference quotient
- Continuous functions, definition of continuity, identifying continuous functions and discontinuities from graphs, continuity properties (p.153), asymptotes and continuity, drawing a graph with certain limit and continuity properties

Here are some problems to help in your studies. Note that when you solve the problems, it is ok to leave exponents and logarithms in the answer without evaluating a numerical answer when the calculation is out of the realm of quick hand calculation, e.g. $5^{30}$ or $\log_3 23$. However, you should evaluate relatively simple things like $3^2$, $5^{-1}$, $\log_8 1$, $\ln e$, $\log_3 9$, and so on.

1. Given the graph of the function $f(x) = |x|$, sketch the graph of $y = -f(x - 2) + 1$. 

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2. Sketch the graph of

\[ g(x) = \begin{cases} 
3, & x < -2 \\
x^2, & -2 \leq x \leq 1 \\
3 - x, & x > 1 
\end{cases} \]

3. In problems 1.1.7-12 in the textbook, which ones are graphs of functions? Explain your reasoning.

4. Find the equation of the line through the points (3, 4) and (7, -5).

5. The Jar-Jar Boutique caters exclusively to the fans of Jar-Jar Binks. In fact, they sell exactly one kind of item, a Jar-Jar porcelain honey jar (they leave the lids ajar to make them extra cute). They also have bulk discounts if you buy multiple ajar Jar-Jar jars. If a customer buys 10 jars, they cost $100, and if a customer buys 20 jars, they cost $200.

(a) Find a linear equation \( f(x) \) for the cost of your purchase if you buy \( x \) jars.

(b) How much does 1 jar cost?

(c) If you bump any case in which the merchandise is kept and they break, then the shopkeeper will charge you double the purchase price. Find the function \( g(x) \) for the cost of accidentally jarring \( x \) ajar Jar-Jar jars.

(d) If, in addition, you also burn down the shop, the replacement cost will be $20,000 plus triple the original purchase price. How much would it cost for \( x \) charred jarred ajar Jar-Jar jars?

(e) Would you, personally, ever visit this shop? Why or why not?

(P.S. In case you are worried, no such verbal trickery will appear on the actual midterm.)

6. Given the quadratic equation

\[ h(x) = x^2 - 6x + 10, \]

(a) find the \( x \)-intercepts (if they exist),
(b) the \( y \)-intercept,
(c) the coordinates of the vertex,
(d) and sketch the graph of this function.

(e) Without actually finding the \( x \)-intercepts, how can you find out if there are 0, 1, or 2 intercepts?

7. Answer the questions for problems 2.1.7-14
8. What is the most number of roots that this polynomial can have? Why?

\[ p(x) = 3x^7 + \pi x^{32} - x^2 + 17022x^{1001} \]

9. Consider the function

\[ r(x) = \frac{2x - 3}{x + 1}. \]

(a) What are the \( x \)-intercept(s)?
(b) What are the \( y \)-intercept(s)?
(c) Find the horizontal and vertical asymptotes.
(d) Using your information, sketch a graph of this function.

10. Solve the equation

\[ 3^{5x+5} = 9^{3x+1}. \]

11. You invest $1500 in an account which pays 12% interest compounded quarterly. How much will you have after 10 years?

12. Combine so that you express this as a single logarithm:

\[ 2 \log_5 x + \log_5 7 - 3 \log_5 y + 1 \]

13. Evaluate the following expressions

(a) \( \log_3 9 \)
(b) \( \log_4 32 \)
(c) \( \log_5 5^{29} \)
(d) \( 6^{\log_6 7} \)

14. Solve the equations

(a) \( 4^{3x-1} = 50 \)
(b) \( 3^{x+1} = 5^x \)
(c) \( \log_5(x - 1) + \log_5(x + 3) = 1 \)

15. Suppose you invest $2000 in an account paying 12% interest compounded monthly, how long will it take for you to accumulate $9000?
16. If you invest $1500 in an account with continuously compounding interest, and it takes 20 years to get a return $15,000, what is the rate of interest you are getting on the account? (Note this problem is like the problems economists solve when they talk about the average rate of growth of aspects of the economy such as rate of growth of the GDP over the past, say, 20 years.)

17. Using the properties of limits, find
\[ \lim_{x \to 3} \sqrt{4x + 4}. \]

18. Consider the function
\[ f(x) = \frac{x^2 - 9}{x - 3}. \]
(a) Using the properties of limits, find \( \lim_{x \to 3} f(x) \), if it exists.
(b) Find \( f(3) \), if it exists.
(c) Does \( \lim_{x \to 3} f(x) = f(3) \)? If not, can you redefine \( f(3) \) so that it does?

   (a) For which \( x \) values in the domain is this function discontinuous?
   (b) For these \( x \) values you found, for which ones can you define or redefine \( f(x) \) at that point so that the function is continuous.
   (c) For which points \( x = c \) will the function always be discontinuous, regardless of the value of \( f(c) \)?

20. Sketch a graph of a function \( y = g(x) \) which has these properties:
   (a) \( \lim_{x \to -2^-} g(x) = 1 \)
   (b) \( \lim_{x \to -2^+} g(x) = 3 \).
   (c) \( g(2) = -1 \).
   (d) \( g(x) \) is continuous for all \( x \neq 2 \).
   (e) \( g(-2) = 4 \).
   (Note: There are a lot of possible solutions to this question.)