Math 352, Homework Set #3.

1. A rare genetic disease is discovered. Although only one in a million people carry it, you consider getting screened. You are told that the genetic test is extremely good; it is 100% sensitive (it is always correct if you have the disease) and 99.99% specific (it gives a false positive result only 0.01% of the time). Having recently learned Bayes’ Theorem, you decide not to take the test. Why?

2. (#3 from the textbook, p. 85) Three different machines A, B, and C were used for producing a large batch of items. Suppose that machines A, B, and C produce 20%, 30%, and 50% of the items, respectively. Furthermore, suppose that 1%, 2%, and 3% of the items produced by machines A, B, and C, respectively, are defective. If one item is selected at random from the entire batch and is found to be non-defective, what is the (posterior) probability that it was produced by machine B?

3. (#5 from the textbook, p. 85) In a certain city, 30% of the people are Conservatives, 50% are Liberals, and 20% are Independents. Records show that in a particular election, 65% of the Conservatives voted, 82% of the Liberals voted, and 50% of the Independents voted. If a person in the city is selected at random and it is learned that he/she did not vote in the last election, what is the probability that he/she is a Liberal?

4. A house has two alarm systems which operate independently of one another. If no robber is present, then the first system will sound 20% of the time, while the second system will sound 10% of the time. One evening, there is a 40% probability that a robber will strike this house. If both alarm systems are triggered, then what is the probability that the robber is indeed present?

5. Flat fortune cookies are produced by 4 production lines at the same rate, and normally only 5% are defective. The cookies from all 4 lines are dispersed evenly in the outgoing shipments. One day, one of the lines has a technical glitch and now 10% of its cookies are defective. The cookies are shipped before this glitch is discovered, and a customer receives a bag of these flat fortune cookies. He/she randomly picks 3 of these cookies and finds that 1 of them is defective.
   
   (a) What is the probability that a flat fortune cookie is defective? (Use the law of total probability; you will need to use the binomial distribution to compute the conditional probabilities.)
   
   (b) What is the probability that the defective flat fortune cookie is from the problematic production line?

6. Create your own Baye's Rule problem, along with a worked out solution.