Math 352, Homework Set #8.

1. Suppose that in any given year, the slugging percentage of all MLB players lie in a Normal Distribution. In 2008, the mean slugging percentage was .416 with a standard deviation of .080.

   (a) Find the probability of a hitter having a slugging percentage of at least .430.
   (b) Find the probability of 16 batters having a mean slugging percentage of at least .430. What about for 64 batters?

2. Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

   (a) Suppose that students scoring in the top 10% of this distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A?
   (b) If it is known that a student’s score exceeds 72, what is the probability that his or her score exceeds 84?

3. A soft drink machine can be regulated so that it discharges an average of \( \mu \) ounces per cup. If the ounces of full are normally distributed with standard deviation 0.3 ounce, give the setting of \( \mu \) so that 8-ounce cups will overflow only 1% of the time.

4. The service times (in minutes) for customers through a checkout counter in a retail store are independent random variables with mean 1.5 and variance 1.0. Approximate the probability that 100 customers can be serviced in less than 2 hours of total service time. Interpret your result.

5. Suppose that \( X \) is a random variable with mean 10 and variance 15. What can we say about \( P(5 < X < 15) \)?

6. Let \( X \) be a random variable defined by \( P(X = \pm 1) = \frac{1}{8}, P(X = 0) = \frac{6}{8} \) and 0 otherwise. Use Chebychev’s Inequality to find a bound for \( P(|X - \mu| \geq 2\sigma) \). Then, find the actual value of \( P(|X - \mu| \geq 2\sigma) \). How do these compare?

7. Let \( X \) be a random variable with mean \( \mu \) and variance \( \sigma^2 \). Construct a probability distribution for \( X \) such that \( P(|X - \mu| \geq 3\sigma) = \frac{1}{5} \).

8. Let \( X_1, X_2, \ldots, X_{10} \) be independent Poisson random variables with mean 1.
   
   (a) Use the Markov inequality to find a bound for \( P(X_1 + \ldots + X_{10} \geq 15) \).
   
   (b) Use the Central Limit Theorem to approximate \( P(X_1 + \ldots + X_{10} \geq 15) \).

9. Use Chebychev’s Inequality to prove the Weak Law of Large Numbers:
   
   If \( X_1, X_2, \ldots, X_n \) are independent identically distributed RVs with mean \( \mu \) and variance \( \sigma^2 \), then for any \( \varepsilon > 0 \), we have
   
   \[
P\left( \left| \frac{X_1 + \ldots + X_n}{n} - \mu \right| > \varepsilon \right) \to 0 \quad \text{as} \quad n \to \infty.
   \]