Math 352, Homework Set #9.

1. (a) If $Y$ has a geometric distribution with probability of success $p$, show that the moment generating function for $Y$ is $m(t) = \frac{pe^t}{1-qe^t}$, where $q = 1 - p$.

(b) Differentiate $m(t)$ from the previous exercise to determine $E[Y]$ and $E[Y^2]$. Then, find Var $Y$.

2. (#8, p. 240 from the textbook) Suppose that $X$ is a random variable for which the moment generating function is given by $m(t) = e^{t^2+3t}$ for all $t \in \mathbb{R}$. Find the mean and variance of $X$.

3. Find the distribution of the random variable $Y$ for each of the following moment generating functions: (a) $\left(\frac{1}{3}e^t + \frac{2}{3}\right)^5$, (b) $\frac{e^t}{2-e^t}$, (c) $e^{2(e^t-1)}$.

4. (#11, p. 240 from the textbook) Suppose that $X$ is a random variable for which the moment generating function is given by $m(t) = \frac{1}{5}e^t + \frac{2}{5}e^{2t} + \frac{2}{5}e^{3t}$ for all $t \in \mathbb{R}$. Find the probability distribution of $X$. Remark: It is a simple discrete distribution.

5. (a) If $Y$ is a random variable with moment generating function $m(t)$, and if $W = aY + b$ for some constants $a$ and $b$, show that the moment generating function of $W$ is $e^{bt}m(at)$.

(b) Deduce $E(W)$ and Var $W$ where $W$ is defined in the previous exercise. Do these results look familiar?

6. Let $r(t) = \ln[m(t)]$, where $m(t)$ is a moment generating function for some distribution with mean $\mu$ and variance $\sigma^2$. Show that $r'(0) = \mu$, and $r''(0) = \sigma^2$. (Hint: $m(0) = 1$.)

7. Let $Y_1, Y_2, ..., Y_n$ be independent normally distributed random variables with mean $\mu_i$ and variance $\sigma_i^2$ for $i = 1, 2, ..., n$. Define $U = \sum_{i=1}^na_iY_i$ for some constants $a_1, ..., a_n$. Show by moment generating functions that $U$ is normally distributed with mean $\sum_{i=1}^na_i\mu_i$ and variance $\sum_{i=1}^na_i^2\sigma_i^2$.

8. **Bonus:** Compute the moment generating functions for the gamma and beta distributions.