Math 511, The Ninth Homework Set.

1. Find $c_1, c_2, c_3$ such that $\int_{-\pi}^{\pi} [x - (c_1 \sin x + c_2 \sin(2x) + c_3 \sin(3x))]^2 \, dx$ is a minimum.

2. Show that $\{\cos(n \arccos x) : n = 0, 1, 2, \ldots\}$ is an orthogonal set on $[-1, 1]$ with respect to weight function $(1 - x^2)^{-1/2}$. (That is, the inner product is given by $\langle f, g \rangle = \int_{-1}^{1} (1 - x^2)^{-1/2} f(x)g(x) \, dx$.) Then, obtain orthonormalise the set.

3. Show that the vector space $X$ of all real-values continuous functions on $[-1, 1]$ is the direct sum of the set of all even continuous functions and the set of all odd continuous functions on $[-1, 1]$.

4. Show that $Y = \{x = (\xi_j) \in l^2(\mathbb{R}) : \xi_{2n} = 0 \text{ for all } n \in \mathbb{N}\}$ is a closed subspace of $l^2(\mathbb{R})$. Then, find $Y^\perp$.

5. Consider the inner product space $l_0$ consisting of all inner sequences of real numbers with only finitely many nonzero terms with the inner product of $l^2(\mathbb{R})$. Let $a = (1, \frac{1}{2}, \frac{1}{3}, \ldots)$.

(a) Show that $W = \{x \in l_0 : \langle x, a \rangle = 0 \text{ in } l^2\}$ is a closed subspace of $l_0$.

(b) Show that $W^\perp = \{0\}$, where $0$ denotes the sequence with all terms 0.

(c) Is $l_0 = W \oplus W^\perp$? Comment on your answer.