Several interesting phenomena have been observed in the study of traffic flows, including traveling, shock and rarefaction waves. In this project we will begin by observing that under some simplifying assumptions Burger’s equation can be used to model traffic flow. We will study traffic flow by looking at the main variables as functions of position \( x \in \mathbb{R} \) and time \( t \geq 0 \), namely, traffic velocity, traffic density and traffic flow (flux), denoted respectively by \( u \), \( \rho \), and \( \phi \). Initially, we study the simple case where we want to derive a continuum model for traffic flow on a single lane of traffic where passing a car is not allowed. Under these assumptions, we arrive at the inviscid Burger’s Equation:

\[
\frac{du}{dt} + uu_x = \frac{\partial}{\partial x} \left( \frac{u^2}{2} \right) = 0,
\]

with the initial condition \( u(x, 0) = u_0(x) \).

In addition to deriving the so-called conservation of cars equation, the aim of this project is to numerically simulate traffic flow under different conditions and compare and contrast some Lax-Friedrichs types of approximations. The second goal of this project is to study the case where there is a traffic interruption such as a traffic light and add artificial viscosity to study the viscous Burger’s equation

\[
\frac{du}{dt} + \frac{\partial}{\partial x} (u^2) = \epsilon u_{xx},
\]

where \( \epsilon \) denotes the viscosity term. If time permits, we will study the associated control problem to the Burger’s equation and how it applies to traffic flow.

This project is appropriate for students that have successfully completed differential, integral and vector calculus. Some knowledge of linear systems and programming in MATLAB is desired, although not necessary. With the guidance of the faculty advisor, students will be enriched in the mathematical theory behind the numerical simulations, will be given enrichment in MATLAB programming and will co-author a manuscript of their findings.

References