Q: Consider the following grammar (Grammar 1).

Non-terminals: \{ S, X, Y \}
Terminals: \{ a \}
Start-symbol: S
Rules:
\[
S \rightarrow XT \mid Y \\
T \rightarrow aa \mid aaT \\
X \rightarrow aa \mid aaX \\
Y \rightarrow aaa \mid aaaY
\]

(a) Is this grammar ambiguous? That is, is there a string that can be derived from the start symbol in more than one way? If so, give an example and show two different derivations.

(b) Consider the following grammar (Grammar 2)

Non-terminals: \{ S \}
Terminals: \{ a \}
Start-symbol: S
Rules:
\[
S \rightarrow aaS \mid aaaS \mid aa
\]

Does it generate the same language as Grammar 1? If not, give an example of a string that can be generated by one grammar but not the other.
A:  (a) Yes. For example

   $S \rightarrow Y \rightarrow aaaY \rightarrow aaaaa$
   $S \rightarrow XT \rightarrow aaT \rightarrow aaaaT \rightarrow aaaaa$

(b) No. For example, Grammar 2 can generate $aa$